

Process Intensification and Green Chemistry

The effect of mixing on chemical reactions

EPFL

Master of Science in Chemical Engineering and Biotechnology

Dr. H. Randall

Rev. 1 04/2024

Content

- Introduction
 - Simultaneous mixing and chemical reaction, mixing classes, segregation
- Effect of complete segregation on reactor performance for simple reaction $A_1 \xrightarrow{k c_1^n} P$
 - Effect of n , effect of RTD
- Effect of segregation intensity on second order reactions $A_1 + A_2 \xrightarrow{k c_1 c_2} P$
 - Effect of segregation intensity on conversion in CSTR and PFR
 - Fully segregated CSTR with premixed feed
- Effect of segregation intensity on performance and selectivity of consecutive-competitive reactions
 - Effect of segregation intensity on selectivity and conversion
- Experimental mixing time characterization
 - Physical methods
 - Chemical methods (consecutive-competitive, parallel-competitive, examples, Villermoux-Dushman reaction)
 - IEM mixing model

Introduction

Simultaneous mixing and chemical reaction

1. Fast mixing - slow reactions

- If mixing time t_{mx} much smaller than reaction time t_r (i.e., reaction rate \ll mixing rate):
 - Mixing and reaction can be considered to take place consecutively
 - Concentration field is homogeneous throughout reactor
 - No further effect of mixing on the transformation rate (performance)
 - No further effect of mixing on the product distribution (selectivity)

Simultaneous mixing and chemical reaction

2. Slow mixing - fast reactions

- If mixing time t_{mx} comparable or longer than reaction time t_r (i.e., reaction rate \geq mixing rate):
 - Mixing and reaction take place simultaneously, not consecutively
 - Reaction medium is inhomogeneous, each zone having different reaction rates
 - Expected impact on the transformation rate (performance)
 - Expected effect on the product distribution (selectivity) in case of complex reaction networks
- To avoid any effect of mixing on the process, t_{mx} should be at least 10 times lower than t_r

Mixing classes

- **Macromixing**

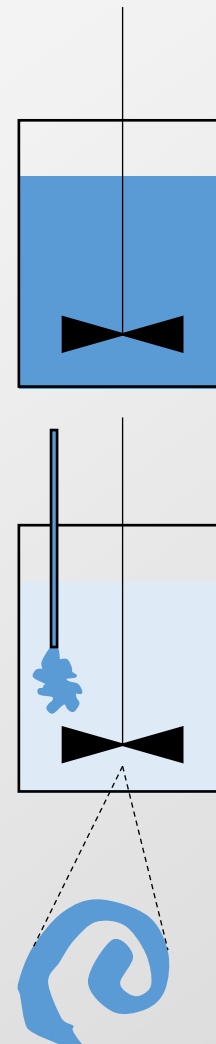
- Distribution of one fluid through the other and equalization of average composition without decreasing local concentration variations

- **Mesomixing**

- Reduction of size of regions with uniform composition
- Increasing of contact between regions of different compositions
- Ex: Feed rate \gg local mixing rate \rightarrow plume

- **Micromixing**

- Mixing at molecular level
- Controls chemical reaction as molecules must physically meet in order to react



- J. Baldyga, R. Pohorechi, Turbulent micromixing in chemical reactors - a review, The Chemical Engineering Journal 58 (1995) 183-195
- John R. Bourne, Mixing and the Selectivity of Chemical Reactions, Organic Process Research & Development 2003, 7, 471-508
- E. L. Paul, V. A. Atiemo-Obeng and S. M. Kresta (Eds.), Handbook of Industrial Mixing: Science and Practice, 2004 Wiley & Sons

Macromixing

- “Distributive” mixing, occurs at the **macroscopic level** (scale of the vessel or reactor)
- It is another term for “blending” to a degree of homogeneity throughout a vessel
- Distribution of one fluid through the other and equalization of average composition without decreasing local concentration variations
- Achieved by **bulk motion** or convective transport until uniform distribution of fluid elements in the reactor volume
- Influences the RTD in a continuous reactor
- Example: mean circulation time in mechanically stirred reactor

$$\tau_c = \frac{V}{C_D N D_i^3} \quad \text{with} \quad 0.7 < C_D < 1$$

Mesomixing

Coarse-scale dispersive mixing via turbulent eddies

Two different mechanisms:

1. **Turbulent dispersion** of fresh fluid element introduced to the reactor, mixing with local surroundings

$$\tau_D = \frac{Q_f}{u D_{turb}} (r_{pipe} \ll L_D) \quad \text{or} \quad \tau_D = \frac{r_{pipe}^2}{u D_{turb}} (r_{pipe} \geq L_D)$$

$$D_{turb} = 0.12 \varepsilon^{1/3} L_D^{4/3} \quad L_D: \text{characteristic length for dispersion}$$

2. **Inertial-convective breakup of large eddies (Eddy disintegration)** (> Kolmogorov scale)

Inertial-convective mesomixing timescale (Baldyga, 1997):

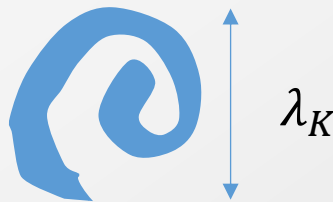
$$\tau_S = A \left(\frac{L_C^2}{\varepsilon} \right)^{1/3} \quad 1 < A < 2 \text{ depending on turbulence level}$$

L_C : concentration macroscale

Micromixing (1)

General definition

- The final stage of mixing, occurring at much smaller length scales than macro- and mesomixing
- Occurs at the scale of smallest turbulent eddies and concentration striations



- Can be characterized e.g., by Kolmogorov and Batchelor scales

Micromixing (2)

Kolmogorov length scale

- Sizes of eddies range from largest integral scales of motion (e.g., close to impeller width, feed pipe diameter or tubular reactor diameter) down to the smallest scale of turbulence.
- Energy is transferred from large eddies to smaller ones (energy cascade) until the smallest scale of turbulence is reached (Kolmogorov scale):
 - At that scale, the local Reynolds number $Re = \frac{u \cdot \lambda_K}{\nu} = 1$
- Below that length scale, viscosity dominates, and turbulent energy is dissipated into heat.
- **Size of smallest eddy:** $\lambda_K = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$
- Time to dissipate eddy of size λ_K : $\tau_K = \frac{u^2}{\varepsilon} = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$
- **Time for mixing** by molecular diffusion within Kolmogorov scale:

$$\tau_K = \frac{\lambda_K^2}{D} = Sc \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

Micromixing (3)

Batchelor length scale

- Smallest length scale of concentration (or temperature) fluctuations.
- Below that scale, molecular diffusion dominates.
- Size of the smallest element of fluid that can homogenize by molecular diffusion in one Kolmogorov time scale ($\tau_B = \tau_K$)
- Size of smallest eddy: $\lambda_B = (\tau_K D)^{1/2} = \left(\frac{\nu D^2}{\varepsilon}\right)^{1/4} = \frac{\lambda_K}{\sqrt{Sc}}$
 - For liquids ($Sc \gg 1$): $\lambda_B \ll \lambda_K \Rightarrow$ Smallest striation thickness given by the Batchelor scale
- Time to dissipate eddy of size λ_B : $\tau_B = \frac{\lambda_B^2}{D} = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$

Micromixing (4)

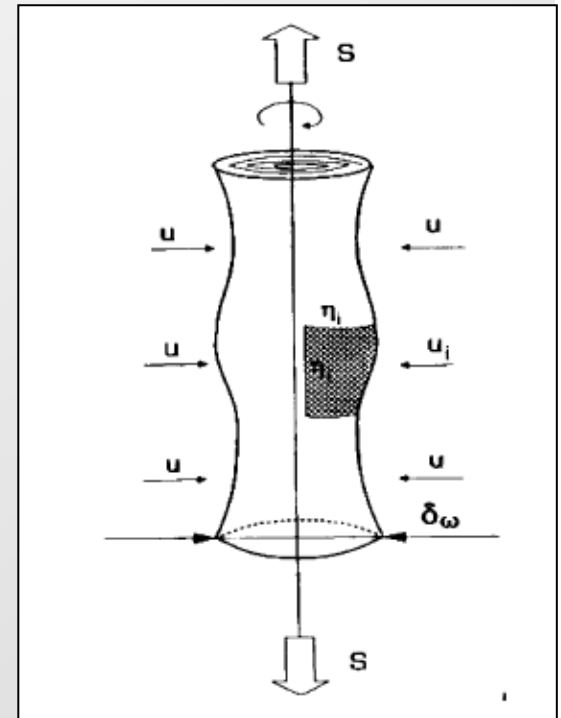
More recent developments: engulfment

- Vortex tubes form in turbulent flow (high Re)
- Micromixing mechanism: small vortices (close to Kolmogorov scale λ_K) draw in (engulf) the surrounding fluid
- Characteristic time for micromixing by engulfment:

$$t_E = 17.2 \sqrt{\frac{\nu}{\varepsilon}}$$

- Rapid micromixing occurs at high energy dissipation / low viscosity

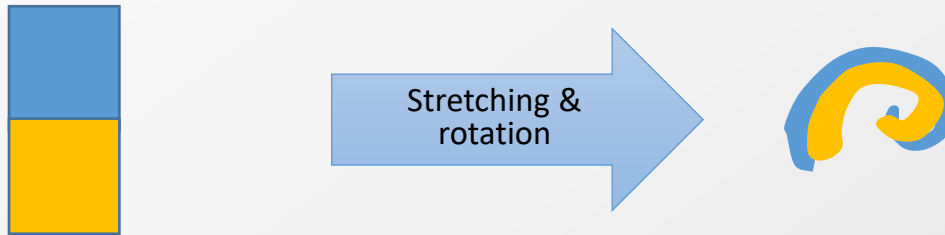
J. Baldyga, R. Pohorechi, The Chemical Engineering Journal 58 (1995) 183-195



Micromixing (5)

More recent developments: shear elongation & diffusion*

- The existence of a shear field leads to stretching and rotation of fluid lamellae



- Half-life of molecular diffusion within progressively thinning lamellae:

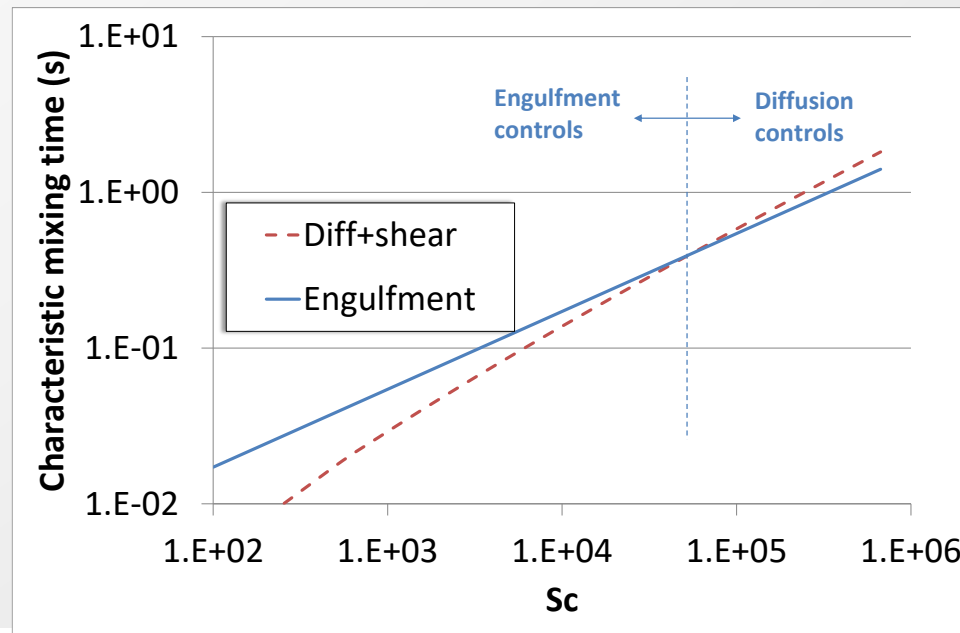
$$\tau_{Ds} \cong 2 \sqrt{\frac{\nu}{\epsilon}} \operatorname{arc} \sinh(0.05 Sc)$$

*See chapter 7

Micromixing (6)

Time constant for engulfment vs stretching and diffusion

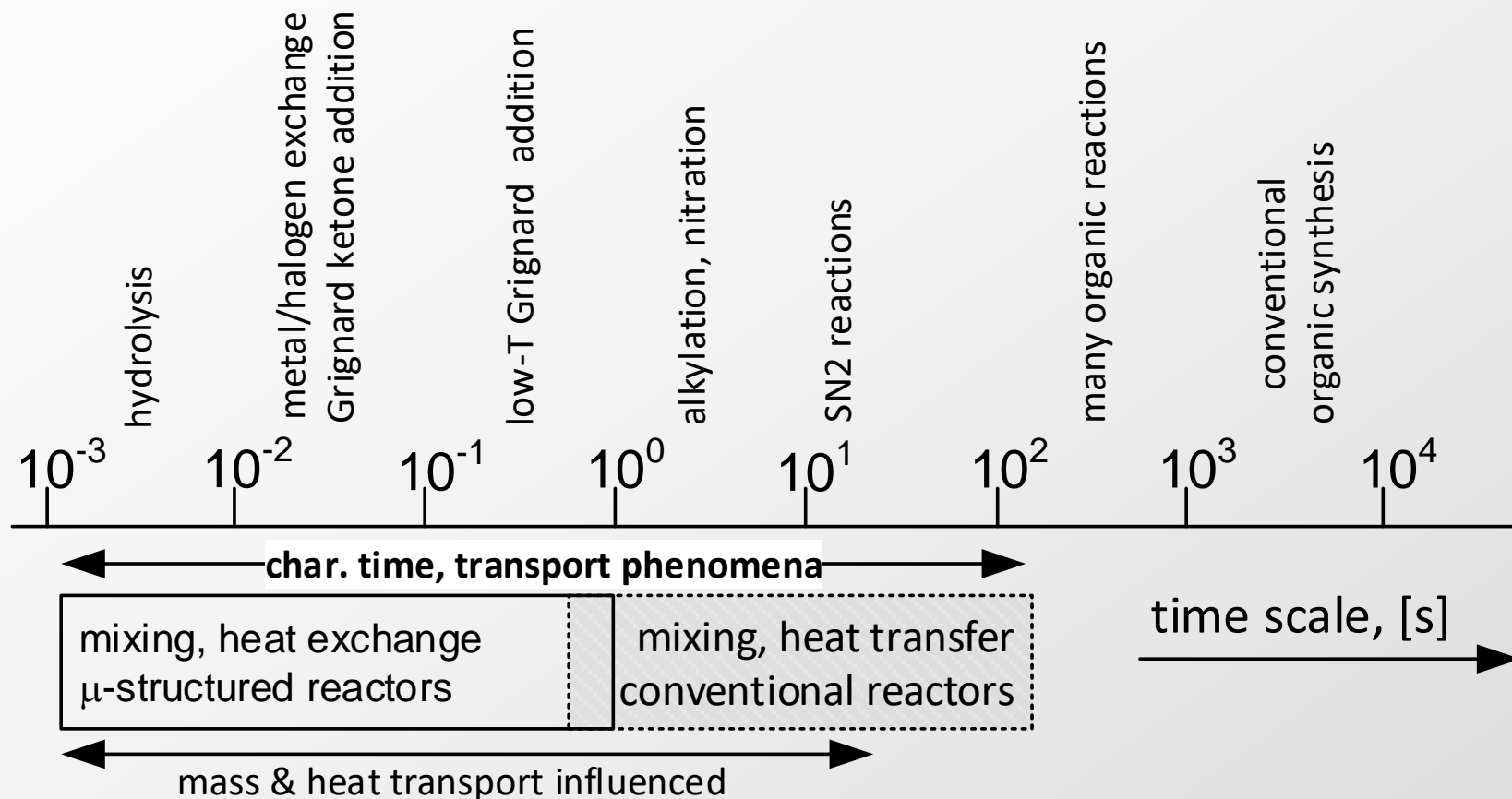
- $Sc = \left[\frac{\text{Rate of motion spreading}}{\text{Rate of concentration smoothing by diffusion}} \right]^2 = \left[\frac{(vt)^{1/2}}{(Dt)^{1/2}} \right]^2 = \frac{\nu}{D}$
- Liquids: $Sc \sim 1000$
- At high Sc number (high viscosity), diffusion controls micromixing rate
- At low Sc number (low viscosity), engulfment controls micromixing rate



Ranking of mixing time constants

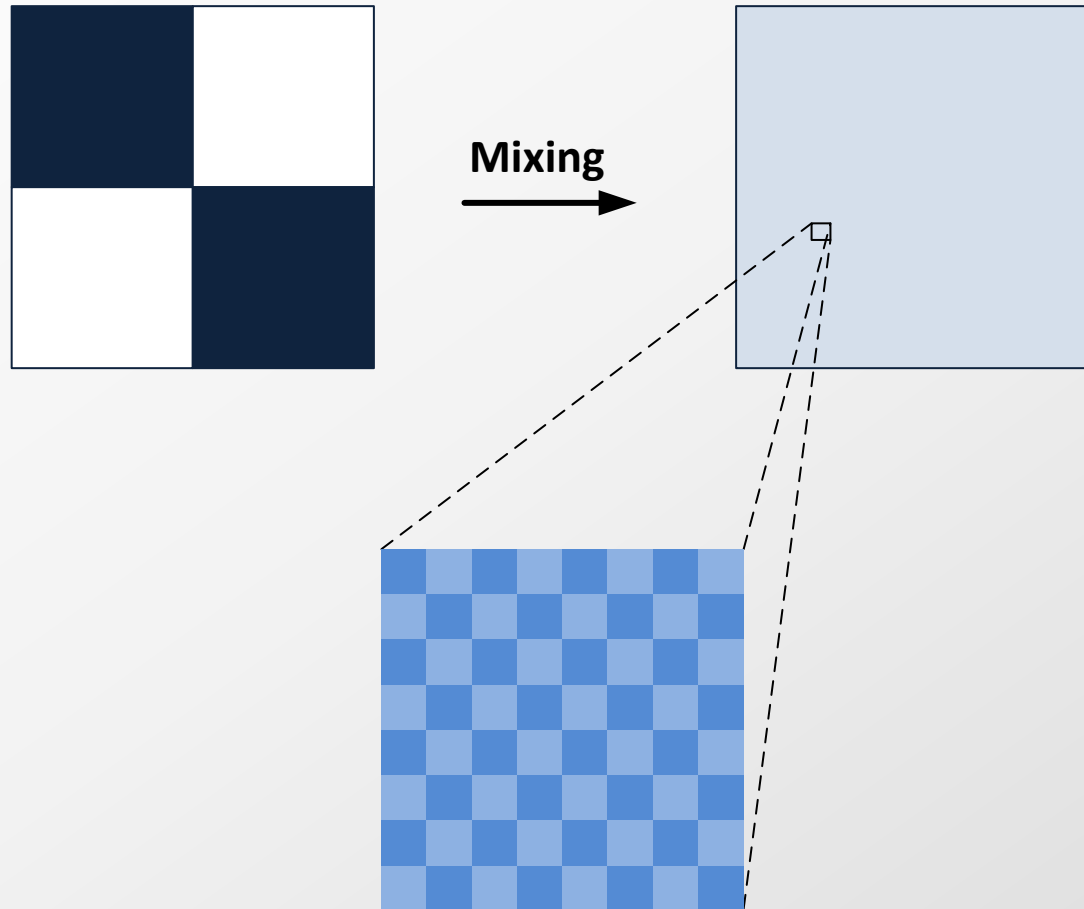
- Three different mixing scales exist, operating with a different mechanism and being characterized by a specific time constant
- The largest time constant is likely to determine the product distribution of fast multiple reactions
- Fast reactions in turbulent environment: meso- and micromixing times often smaller than macromixing time
→ reaction zone located close to feed point (does not spread to the entire vessel)
- The relevant (controlling) mixing mechanism can change with scale!
 - Example for semibatch reactor:
 - Small tank size / low feed rate → likely micromixing control
 - Large tank size / higher feed rate → likely transition to mesomixing control

Examples of characteristic reaction times



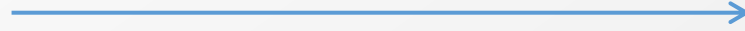
Kashid, M., A. Renken, and L. Kiwi, *Microstructured devices for chemical processing*. 2015: Wiley-VCH.

Segregation

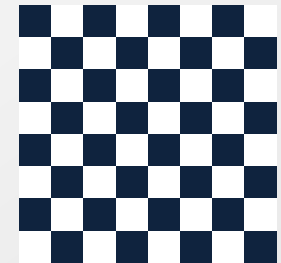
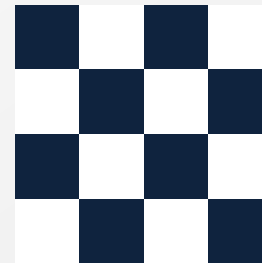
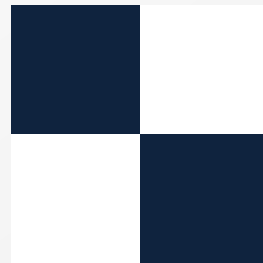


Segregation

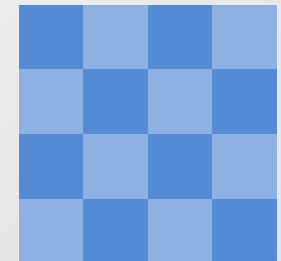
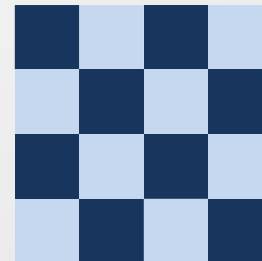
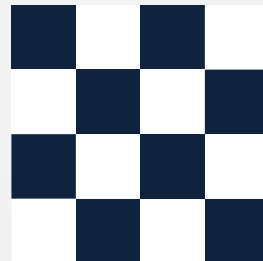
Process time



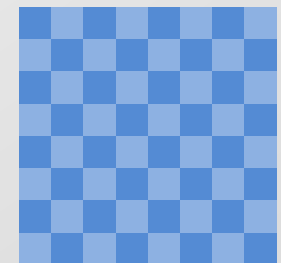
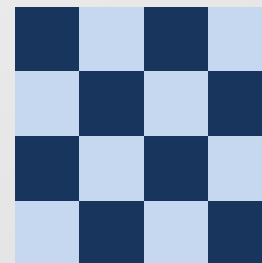
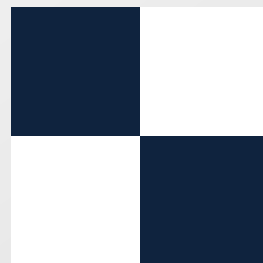
Reduction in
segregation scale:



Reduction in
segregation intensity:



Reduction in
segregation scale and
intensity:



Segregation

- **Scale of segregation**

- Measure of large-scale breakup process (bulk transport and eddy diffusion) without the action of diffusion
- Size of the packets of B that can be distinguished from surrounding fluid A

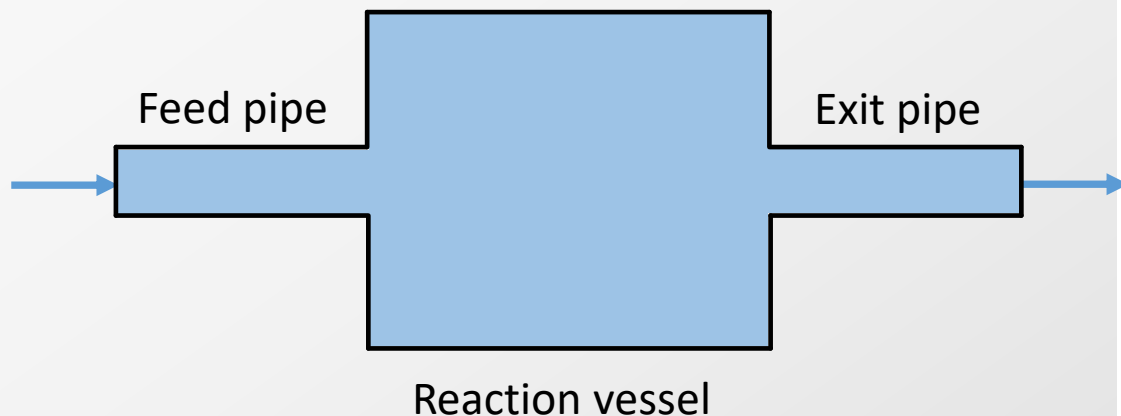
- **Intensity of segregation**

- Measure of difference in concentration between highest-concentration region and lowest concentration region
- Decreases as a result of molecular diffusion
- Does not require any turbulence but turbulence speeds up the mixing process by reducing the size of aggregates thus the diffusion time

Micro-mixed vs completely segregated fluids (2 extreme cases)

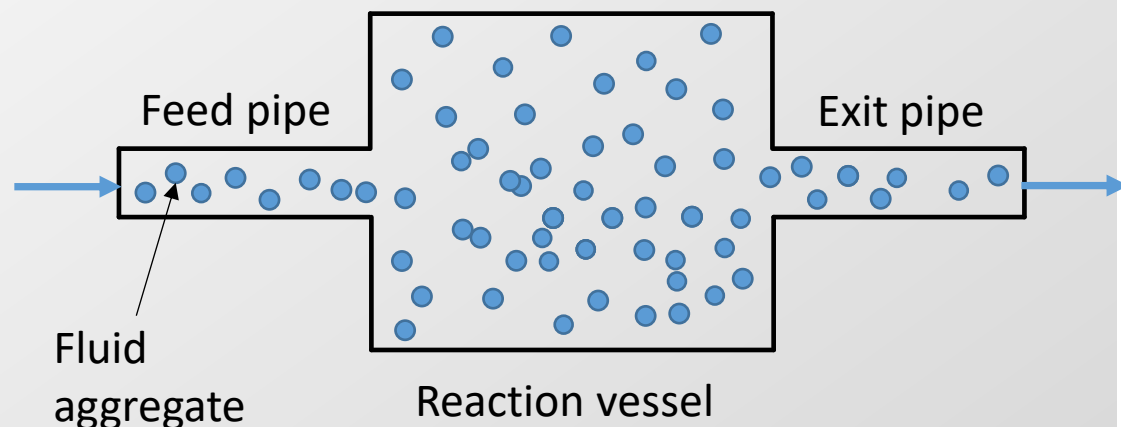
Micro-fluid (premixed feed)

- Micromixed feed
- Individual molecules lose their identity
- Uniform concentration



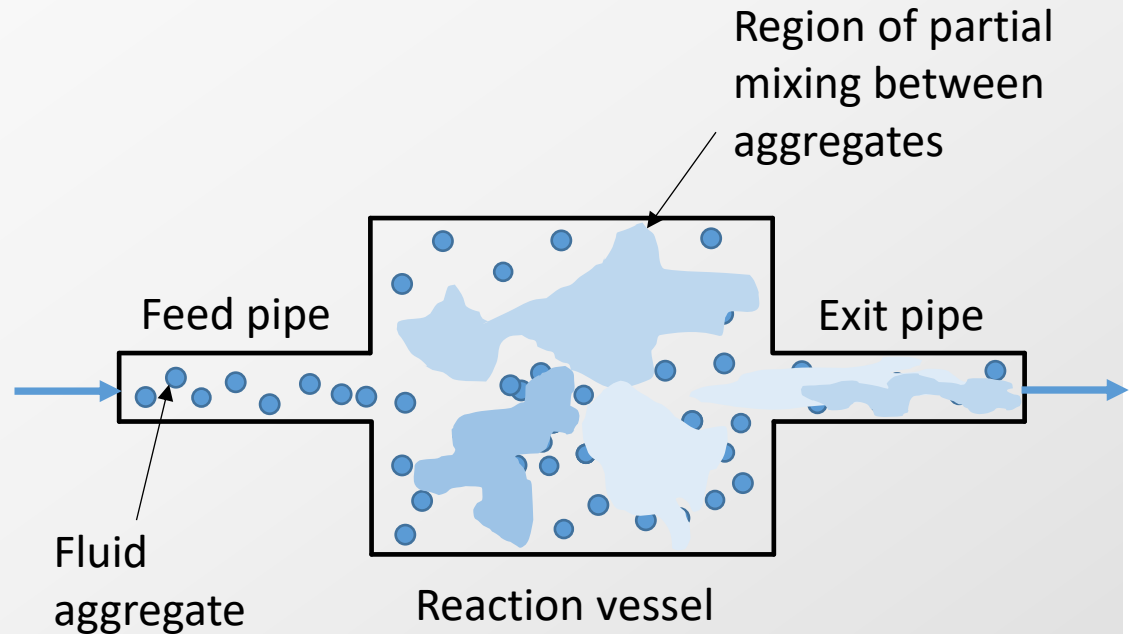
Fully segregated fluid (premixed feed)

- Fully segregated feed
- Fluid aggregates keep their identity (no exchange with other aggregates)
- Concentration differences between aggregates



Partially segregated fluid

- Fully segregated feed
- Fluid aggregates partially lose their identity (partial exchange with other aggregates)
- Concentration differences between regions exist



Segregation in reactors

Segregation in homogeneous systems is a function of the relative values of the following characteristic times

- Characteristic reaction time t_r
- Characteristic mixing time t_{mx}
- Characteristic processing time (ref. time) τ, \bar{t}

Damköhler numbers:

$$DaI = \frac{\tau}{t_r}$$
$$DaII_{mx} = \frac{t_{mx}}{t_r}$$

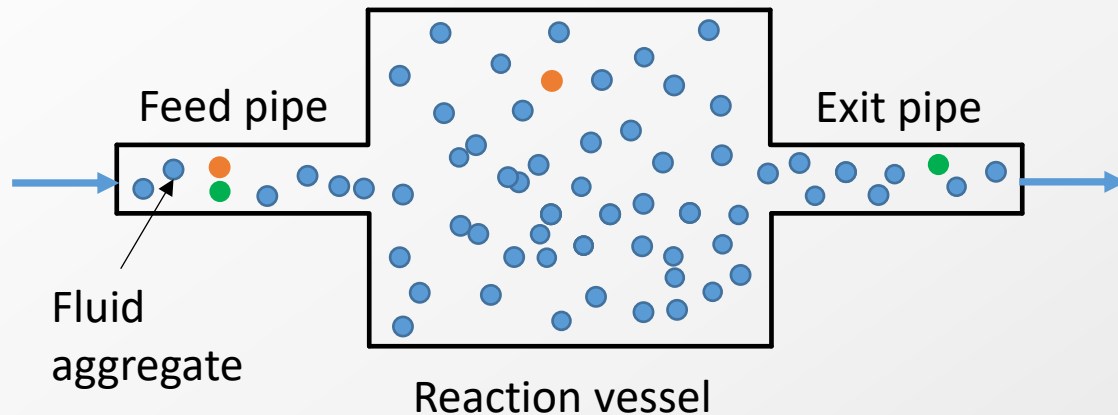
Quality of mixing as a function of the characteristic times

	$t_{mx} \ll t_r$	$t_{mx} \cong t_r$	$t_{mx} \gg t_r$
$t_{mx} \ll \tau$	Micromixed	Partial segregation	Complete segregation
$t_{mx} \cong \tau$	Partial segregation	Partial segregation	Complete segregation
$t_{mx} \gg \tau$	Complete segregation	Complete segregation	Complete segregation

Effect of complete segregation on reactor performance for simple reaction



Conversion in fully segregated fluid



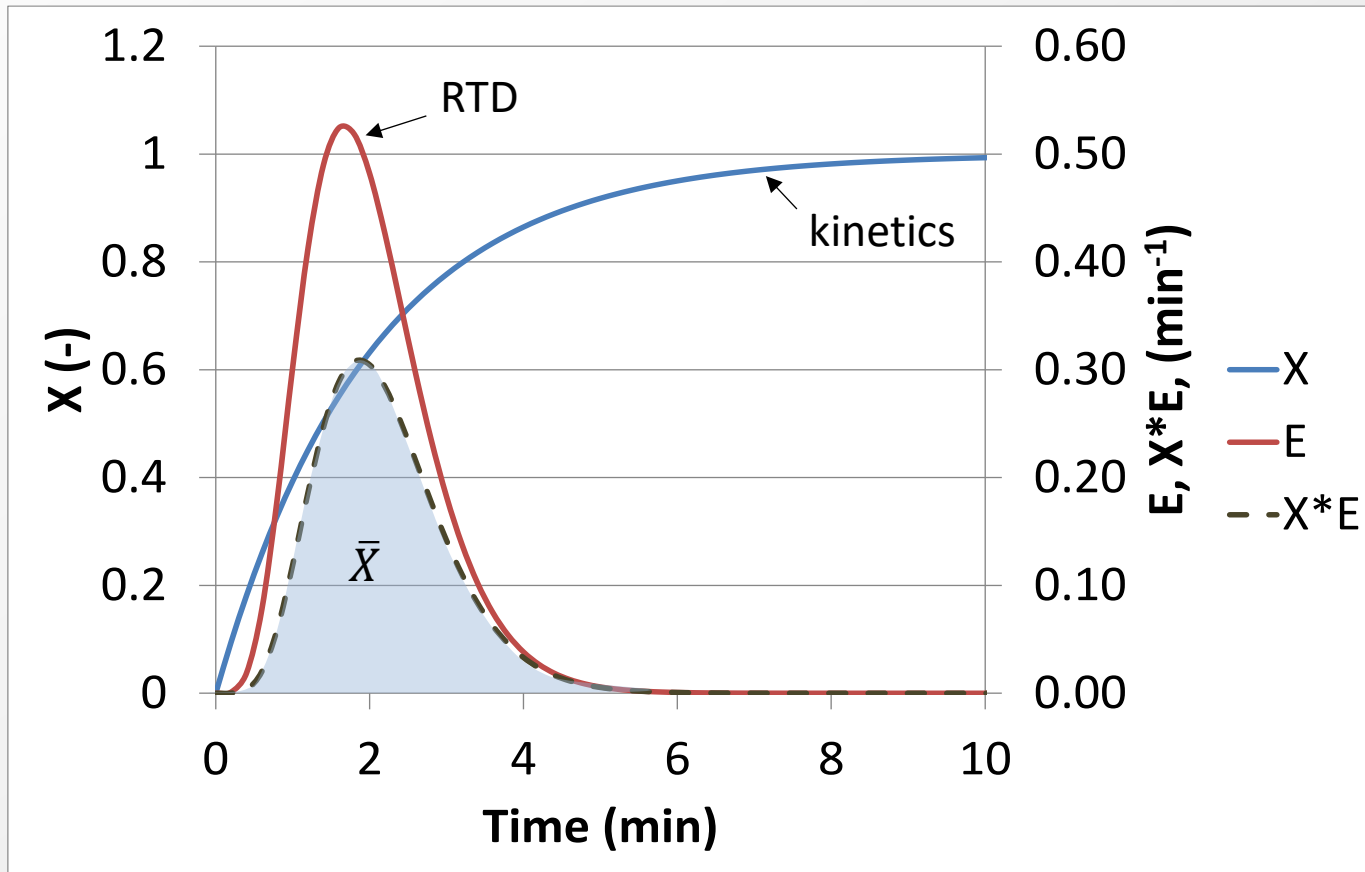
According to the RTD, each aggregate may spend different amounts of time in the reactor but keeps its identity:

Residence-time averaged exit concentration: $\bar{c}_1 = \int_0^{\infty} c_1(t) E(t) dt$

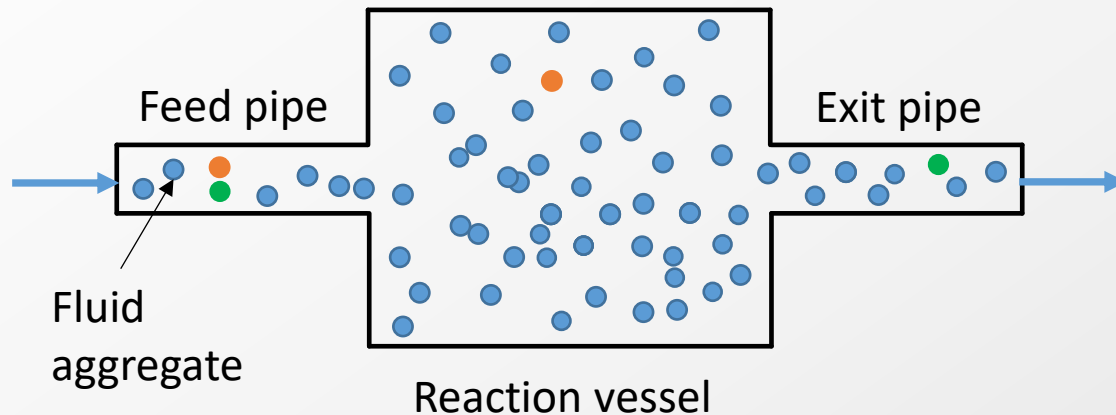
Residence-time averaged exit conversion: $\bar{X} = \int_0^{\infty} \underset{\text{kinetics}}{X(t)} \underset{\text{RTD}}{E(t)} dt$

Conversion in fully segregated fluid

$$\bar{X} = \int_0^{\infty} X(t)E(t)dt$$



Conversion in fully segregated fluid



- Example: conversion for first order reaction in ideal CSTR

$$E(t) = \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right) \quad \text{and} \quad c_1(t) = c_{1,0} \exp(-kt)$$

$$\bar{c}_1 = \int_0^{\infty} c_1(t) E(t) dt = \frac{c_{1,0}}{\tau} \int_0^{\infty} \exp\left[-\left(\frac{1}{\tau} + k\right)t\right] dt = \frac{c_{1,0}}{1 + k\tau}$$

$$\bar{X} = \frac{c_{1,0} - \bar{c}_1}{c_{1,0}} = \frac{k\tau}{1 + k\tau} = \frac{DaI}{1 + DaI} = X(\text{micromixed})$$

⇒ no effect of segregation on conversion for first order processes (see next slide)

Effect of segregation on first order processes

Microfluid

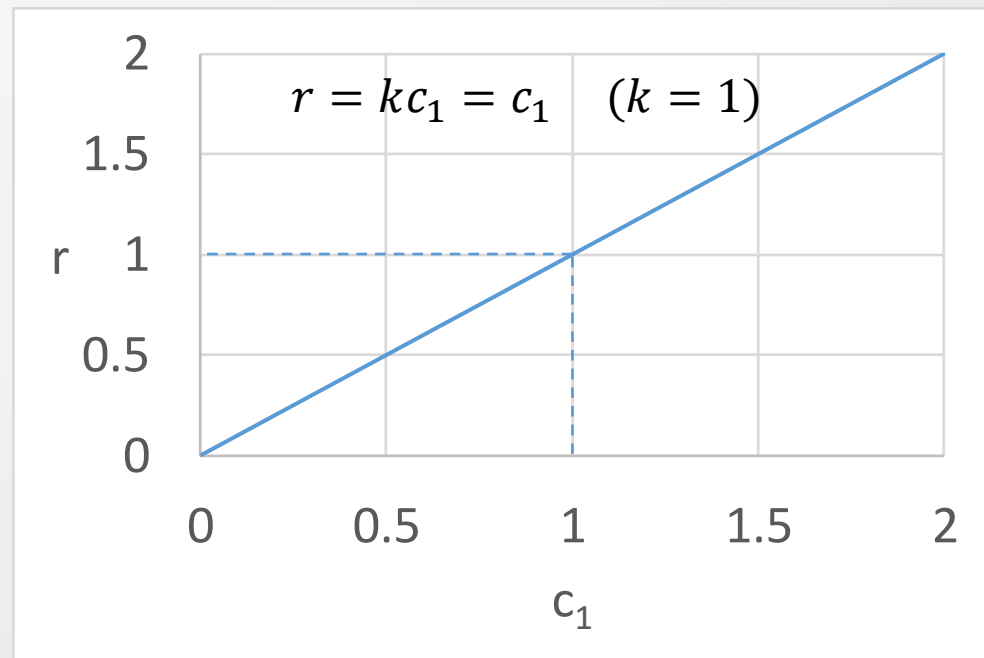
$$c_1 = 1$$
$$r = 1$$

Fully segregated fluid

$$c_{1,1} = 0$$
$$r = 0$$

$$c_{1,2} = 2$$
$$r = 2$$

$$\bar{r} = \frac{0 + 2}{2} = 1 = r(\text{micro})$$



⇒ no effect of segregation on overall reaction rate for 1st order processes

Effect of segregation on second order processes

Microfluid

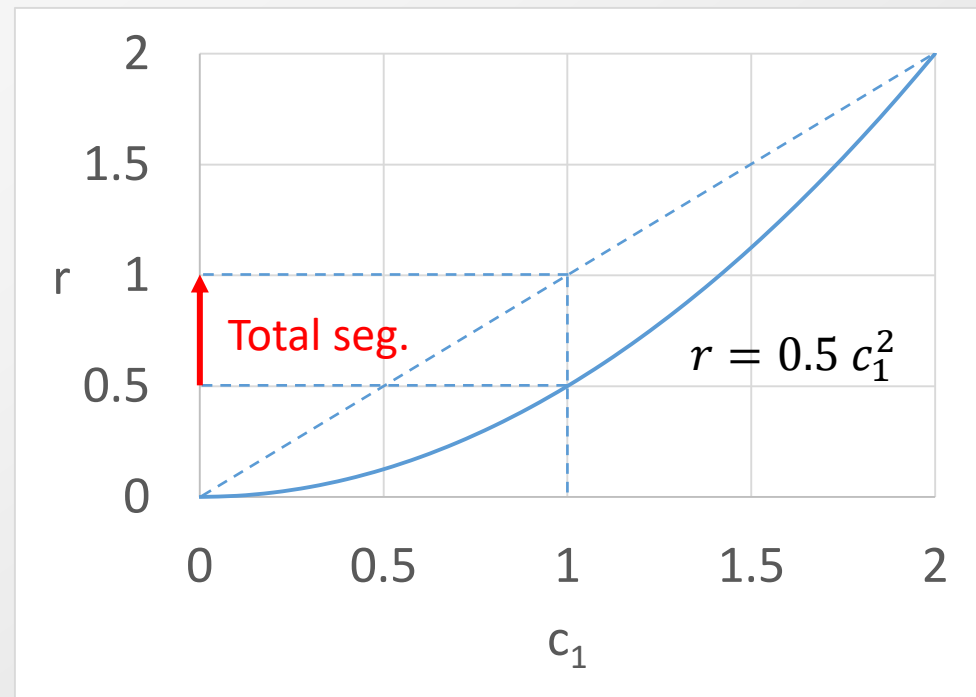
$$c_1 = 1$$

$$r = 0.5$$

Fully segregated fluid

$c_{1,1} = 0$ $r = 0$	$c_{1,2} = 2$ $r = 2$
--------------------------	--------------------------

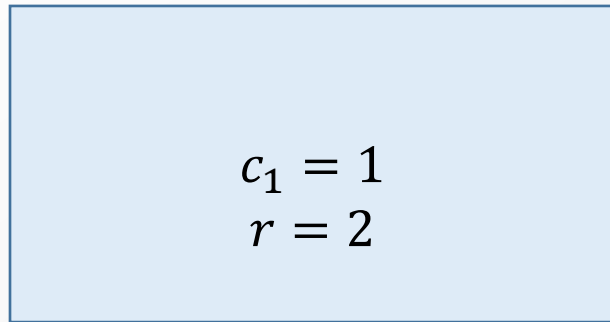
$$\bar{r} = \frac{0 + 2}{2} = 1 > r(\text{micro})$$



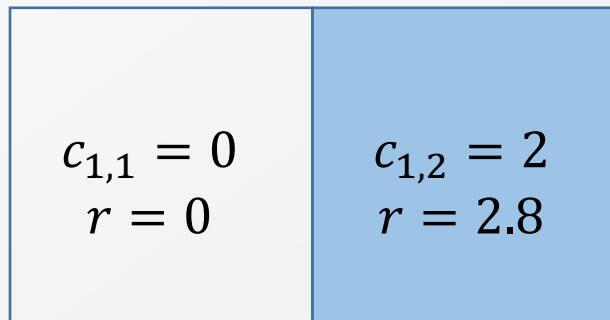
⇒ Positive effect of segregation on overall reaction rate for processes with $n > 1$

Effect of segregation on half-order processes

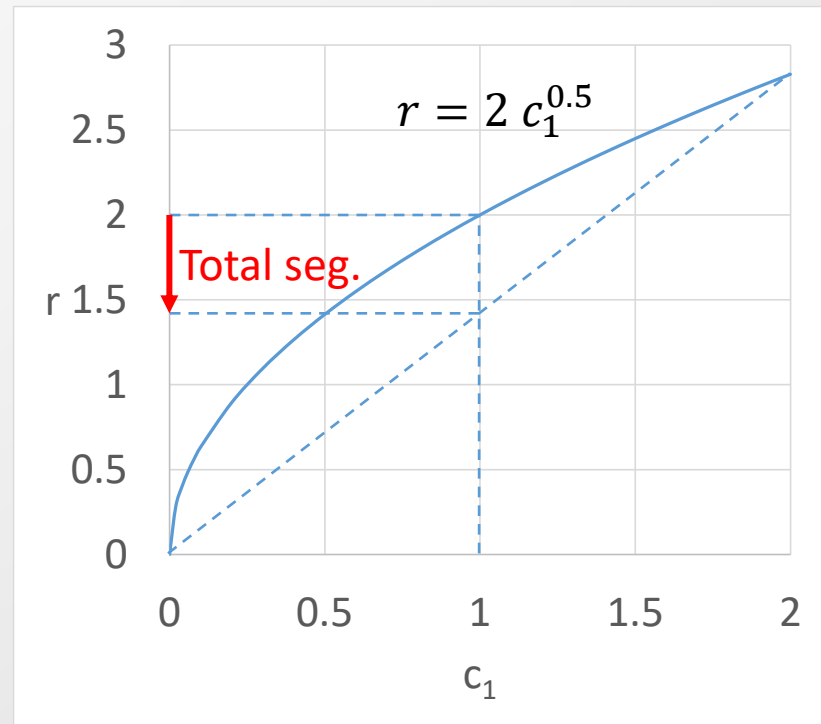
Microfluid



Fully segregated fluid



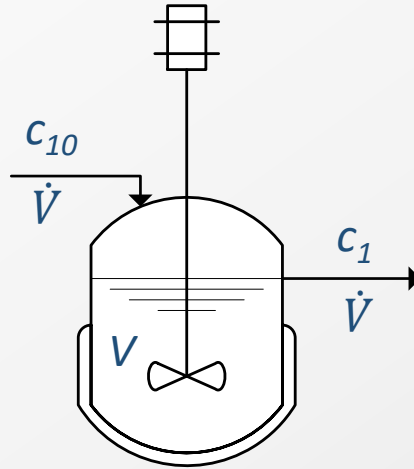
$$\bar{r} = \frac{0 + 2.8}{2} = 1.4 < r(\text{micro})$$



⇒ Negative effect of segregation on overall reaction rate for processes with $n < 1$

Effect of total segregation on conversion

Ideal CSTR, n^{th} order reaction $A_1 \xrightarrow{k} P \quad r = k c_1^n$



$$\tau = \frac{V}{\dot{V}}$$

$$DaI = \frac{\tau}{t_r} = k c_{10}^{n-1} \tau$$

$$X = 1 - \frac{c_1}{c_{10}}$$

Micromixed

$$DaI = \frac{X}{(1 - X)^n}$$

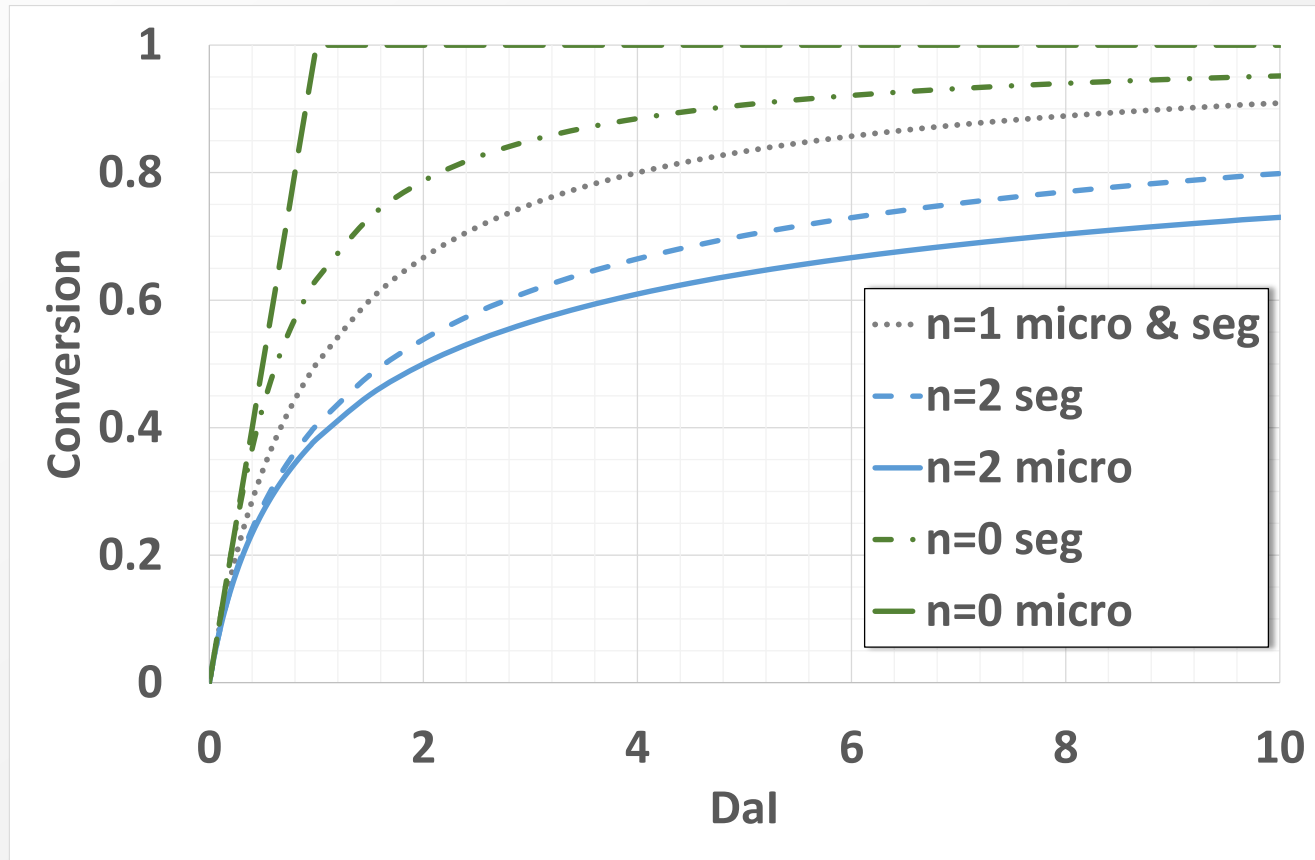
Macromixed (fully segregated)

$$\bar{X} = \frac{1}{\tau} \int_0^{\infty} \left\{ 1 - [1 + (n - 1)k c_{10}^{n-1} t]^{\frac{1}{1-n}} \right\} e^{-\frac{t}{\tau}} dt$$

→ Numerical integration

Effect of total segregation on conversion

Ideal CSTR, n^{th} order reaction



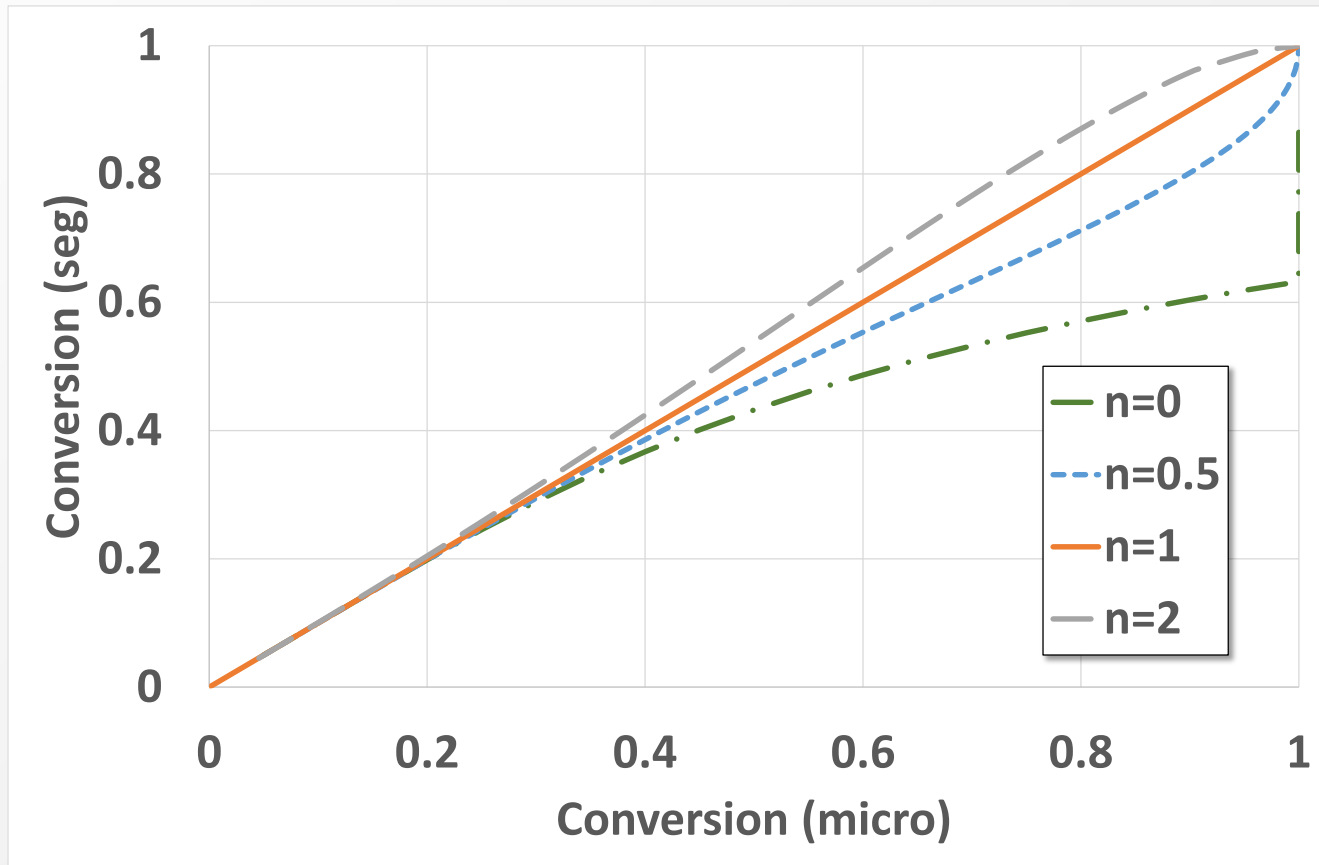
$n > 1 \rightarrow$ positive effect of segregation on performance

$n = 1 \rightarrow$ no effect of segregation on performance

$n < 1 \rightarrow$ negative effect of segregation on performance

Effect of total segregation on conversion

Ideal CSTR, n^{th} order reaction



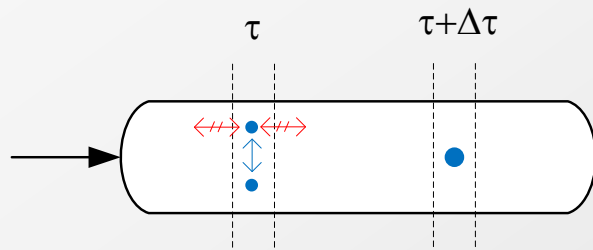
$n > 1 \rightarrow$ positive effect of segregation on performance

$n = 1 \rightarrow$ no effect of segregation on performance

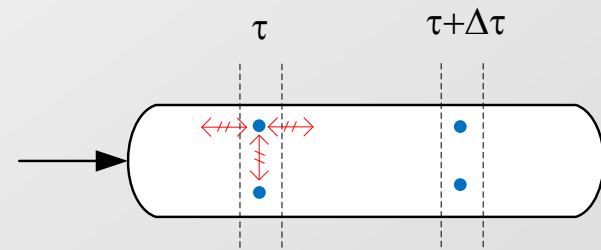
$n < 1 \rightarrow$ negative effect of segregation on performance

Effect of segregation on conversion in PFR

- In PFR, each aggregate or molecule spends same amount of time in reactor
- No mixing between molecules or aggregates of different ages
 - ⇒ Each aggregate behaves as a batch reactor with equal processing time
 - ⇒ No influence of segregation on performance



Plug flow, some degree of mixing (radial)

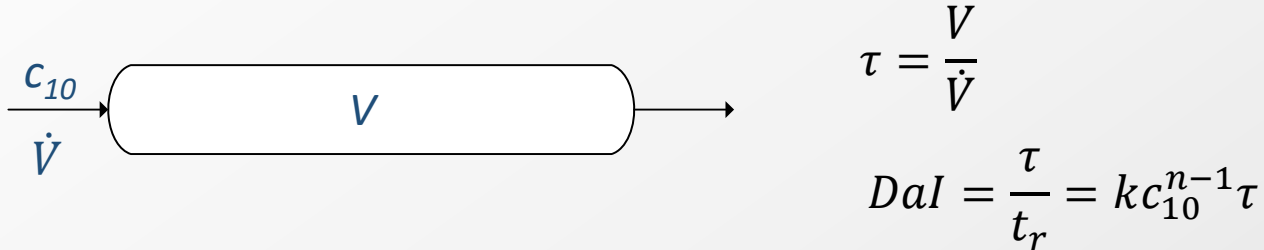


Plug flow, no mixing (full segregation)

⇒ Same result because radial mixing of aggregates with same concentration

PFR: micromixing vs complete segregation

n^{th} order reaction $A_1 \xrightarrow{k} P \quad r = k c_1^n$



Micro and macromixing

$$X = 1 - [1 + (n - 1)DaI]^{\frac{1}{1-n}}$$

Effect of segregation intensity on second order reactions



Segregation characterization

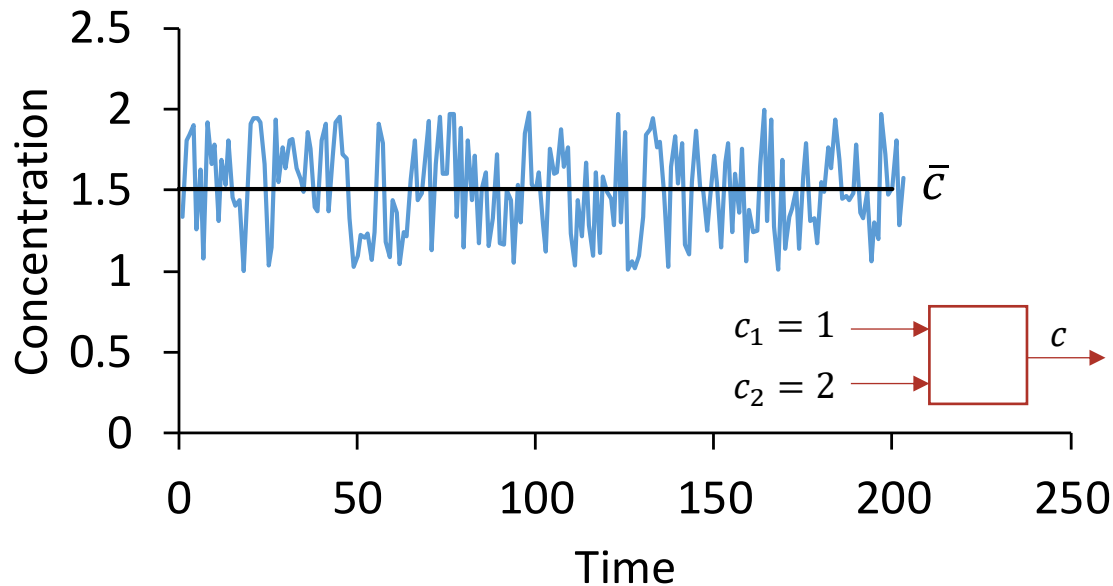
To describe *partial* segregation

- Intensity of segregation in turbulent flow: $I_s = \frac{\sigma^2}{\sigma_0^2}$

$$\sigma^2 = \frac{1}{n} \sum (c - \bar{c})^2 \quad \text{and} \quad \sigma_0^2 = \frac{1}{n} \sum (c - \bar{c}_0)^2$$

Used to measure segregation (“unmixedness”)

At time zero
or in feed



Kinetics of segregation decay

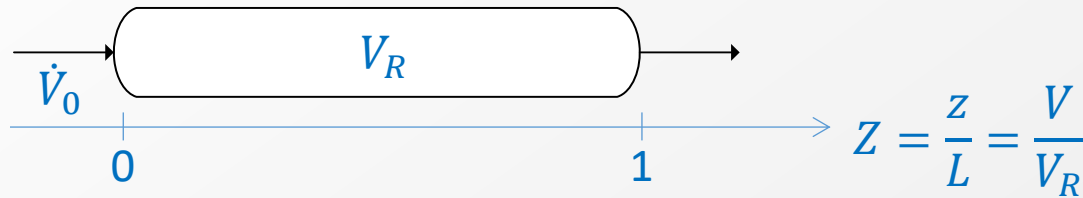
- Assumption: decay of concentration variance σ^2 is a first order process depending upon the intensity of mixing

$$r_d = -b_s \sigma_1^2 = -\frac{1}{t_{mx}} \sigma_1^2$$

b_s [s⁻¹] is a function of the mixing intensity

$$b_s \equiv \frac{1}{t_{mx}}$$

Segregation in PFR



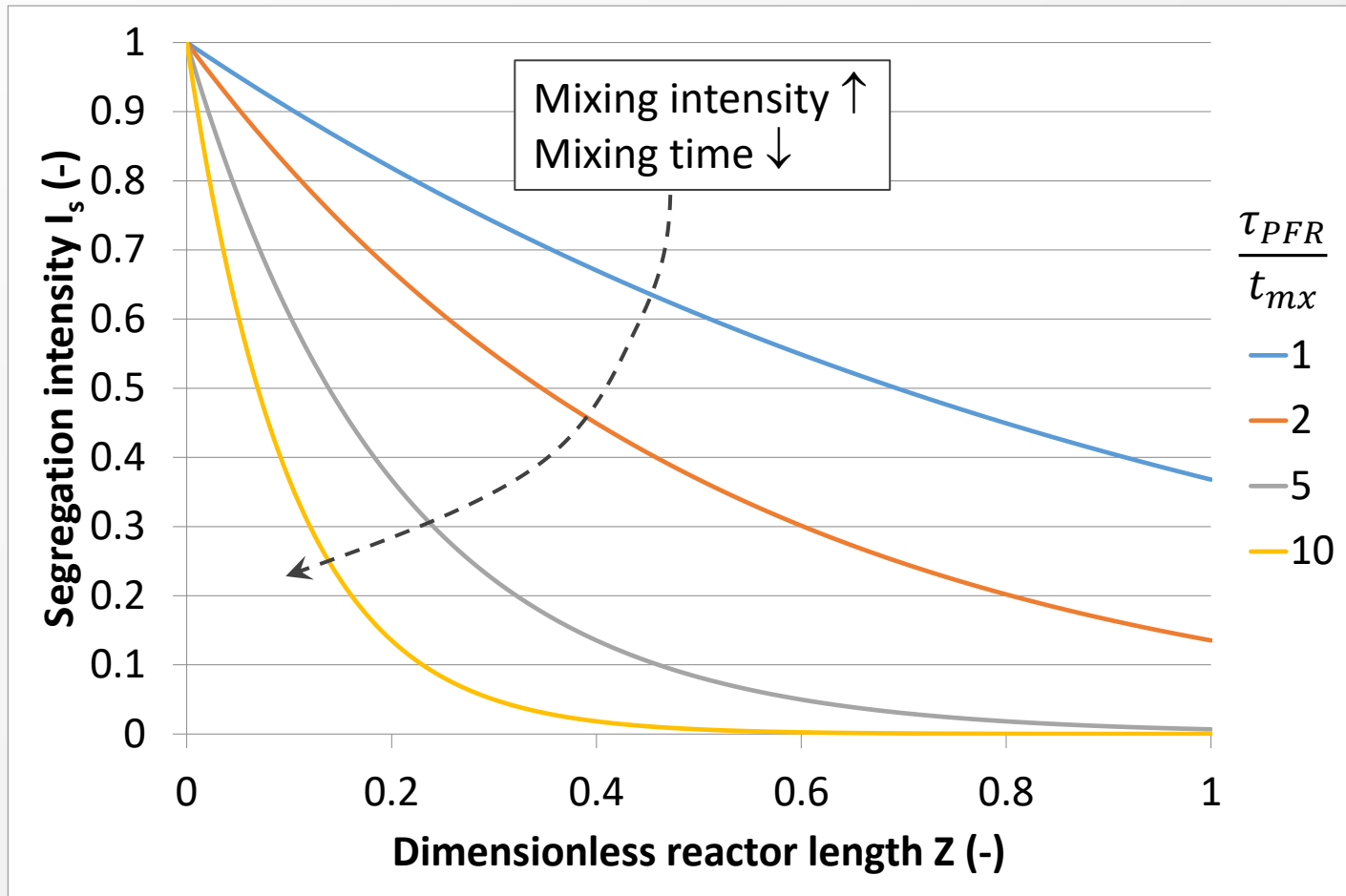
Concentration variance balance: $\frac{d\sigma_1^2}{d\tau} = r_d = -\frac{1}{t_{mx}} \sigma_1^2$

$$I_s = \frac{\sigma_1^2}{\sigma_{1,0}^2} = \exp\left(-\frac{\tau}{t_{mx}}\right) = \exp\left(-\frac{\tau_{PFR}}{t_{mx}} Z\right)$$

where $\tau = \frac{V}{\dot{V}_0}$

$$\text{and } \tau_{PFR} = \frac{V_R}{\dot{V}_0} \quad \tau = Z\tau_{PFR} \quad V = ZV_R$$

Segregation in a PFR

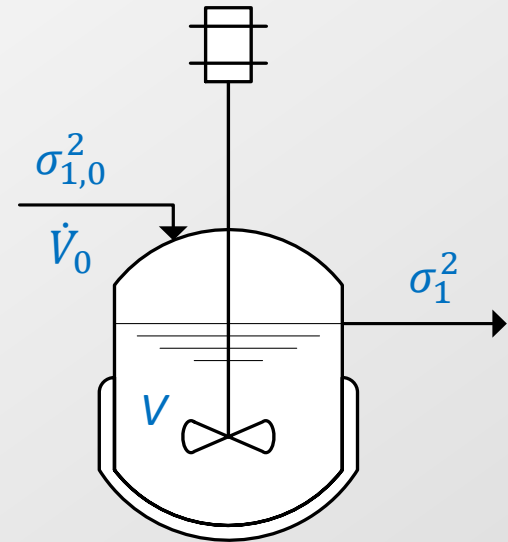


Segregation in an ideal CSTR

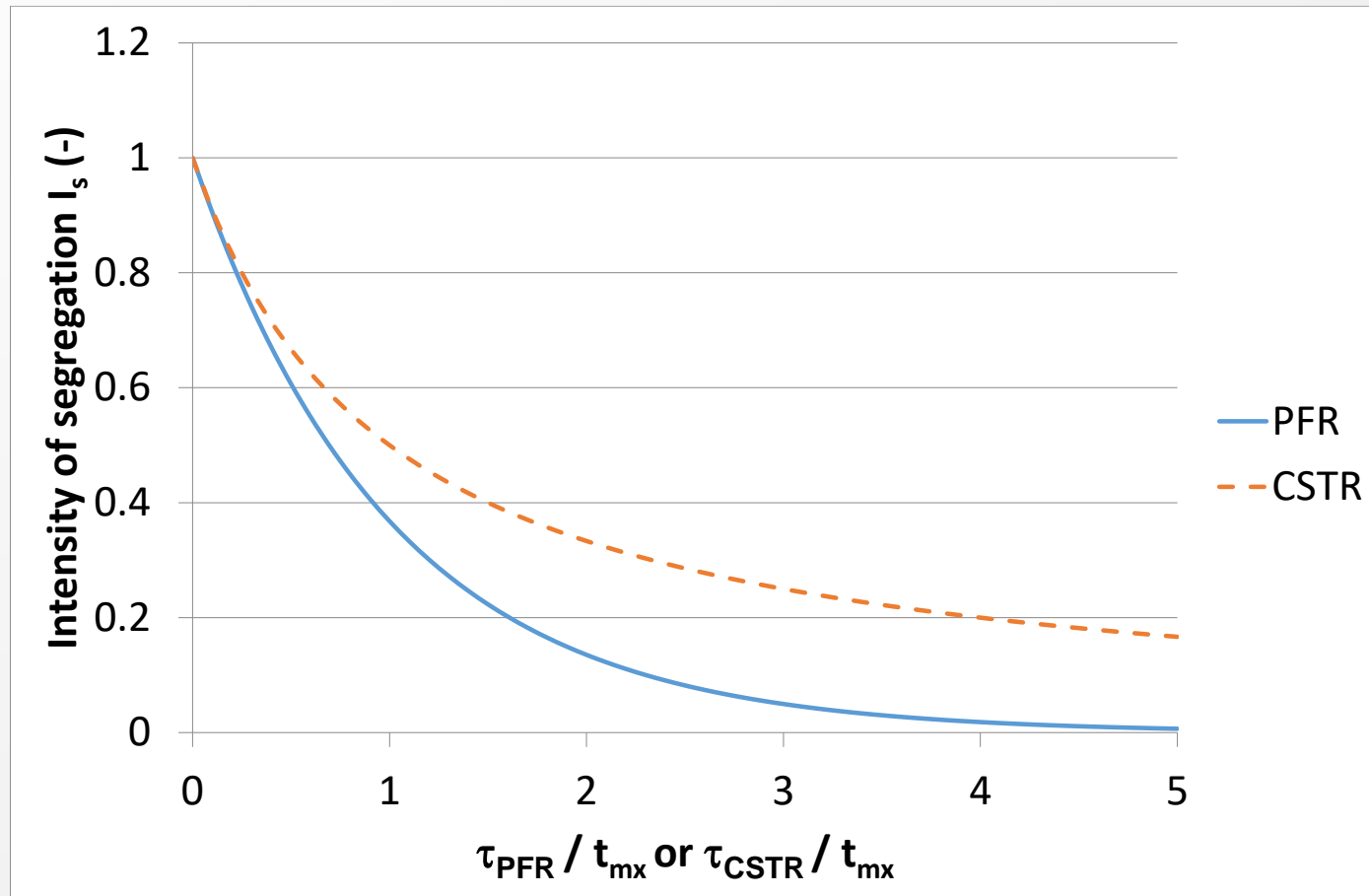
Segregation balance: $\dot{V}_0(\sigma_{1,0}^2 - \sigma_1^2) = -Vr_d = V \frac{\sigma_1^2}{t_{mx}}$

$$I_s = \frac{\sigma_1^2}{\sigma_{1,0}^2} = \frac{1}{1 + \frac{\tau_{CSTR}}{t_{mx}}}$$

where $\tau_{CSTR} = \frac{V}{\dot{V}_0}$



Segregation in ideal CSTR vs PFR



→ Lower segregation in PFR vs ideal CSTR

Effect of segregation intensity on 2nd order bimolecular reactions



Segregation leads to an inhomogeneous reaction mixture

→ Use of a *mean* reaction rate:

$$\bar{R}_1 = -\bar{r} = -k\overline{c_1c_2}$$

$$\overline{c_1c_2} = \bar{c}_1\bar{c}_2 + cov(c_1c_2)^*$$

$$\text{where } cov(c_1c_2) = \frac{1}{n} \sum (c_1 - \bar{c}_1)(c_2 - \bar{c}_2)$$

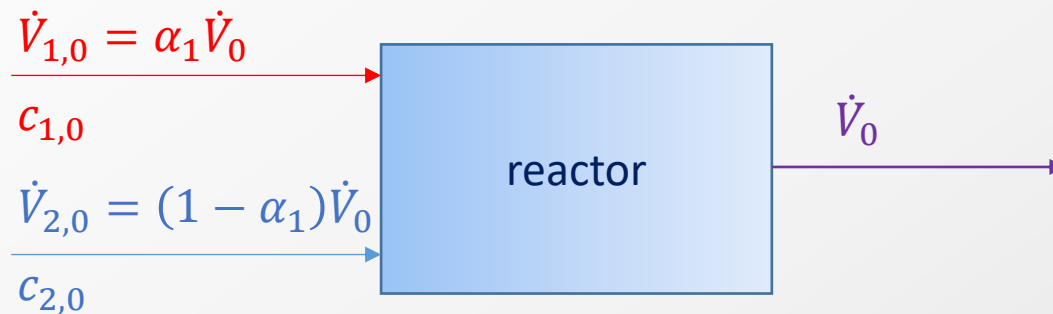
$$\bar{R}_1 = -k[\bar{c}_1\bar{c}_2 + cov(c_1c_2)]$$

*Mean of product = product of means + covariance

Effect of segregation intensity on 2nd order bimolecular reactions

Mixing of species A_1 and A_2

Complete segregation at reactor entrance



$$\alpha_1 = \frac{\dot{V}_{1,0}}{\dot{V}_{1,0} + \dot{V}_{2,0}} = \frac{\dot{V}_{1,0}}{\dot{V}_0}$$

$$\bar{c}_{1,0} = c_{1,0} \alpha_1$$

$$\bar{c}_{2,0} = (1 - \alpha_1) c_{2,0}$$

$$cov(c_1 c_2)_0 = \frac{1}{n} \sum (c_{1,0} - \bar{c}_{1,0}) (c_{2,0} - \bar{c}_{2,0}) = -\bar{c}_{1,0} \bar{c}_{2,0}$$

Effect of segregation intensity on 2nd order bimolecular reactions



Modified rate equation

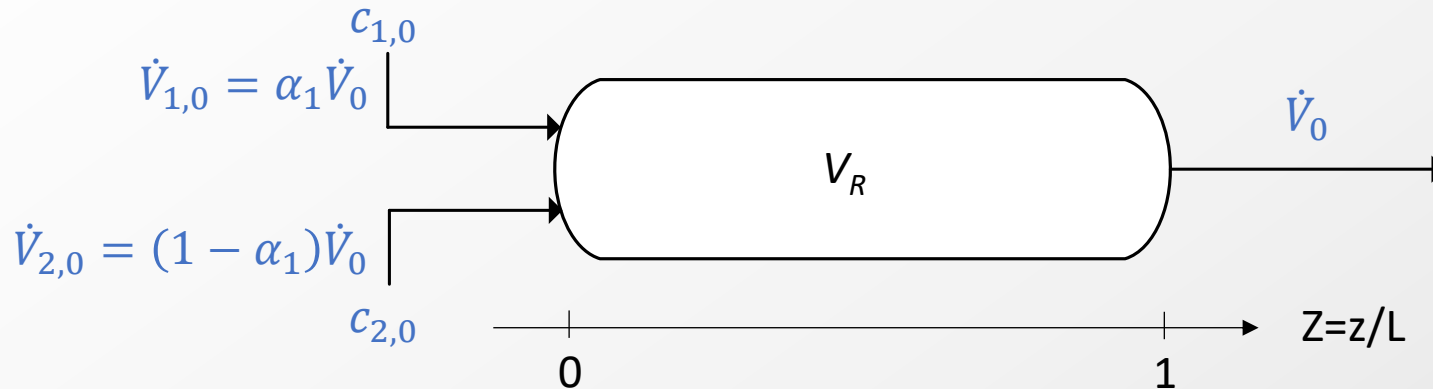
$$\bar{R}_1 = -k(\bar{c}_1\bar{c}_2 + cov(c_1c_2))$$

$$I_s = \frac{cov(c_1c_2)}{cov(c_1c_2)_0}$$

$$cov(c_1c_2) = I_s \cdot cov(c_1c_2)_0 = -I_s \cdot \bar{c}_{1,0}\bar{c}_{2,0}$$

$$\bar{R}_1 = -k(\bar{c}_1\bar{c}_2 - I_s\bar{c}_{1,0}\bar{c}_{2,0}) = -k(\bar{c}_1\bar{c}_2 - I_sc_{1,0}c_{2,0}\alpha_1(1 - \alpha_1))$$

Effect of segregation intensity in PFR



PFR: infinitesimal mass balance for component A_1 :

$$-\frac{d\bar{c}_1}{dZ} = \tau_{PFR} \bar{R}_1 = -\tau_{PFR} k(\bar{c}_1 \bar{c}_2 - I_s \bar{c}_{1,0} \bar{c}_{2,0})$$

$$\frac{df_1}{dZ} = -DaI[f_1(f_1 + M - 1) - I_s M]$$

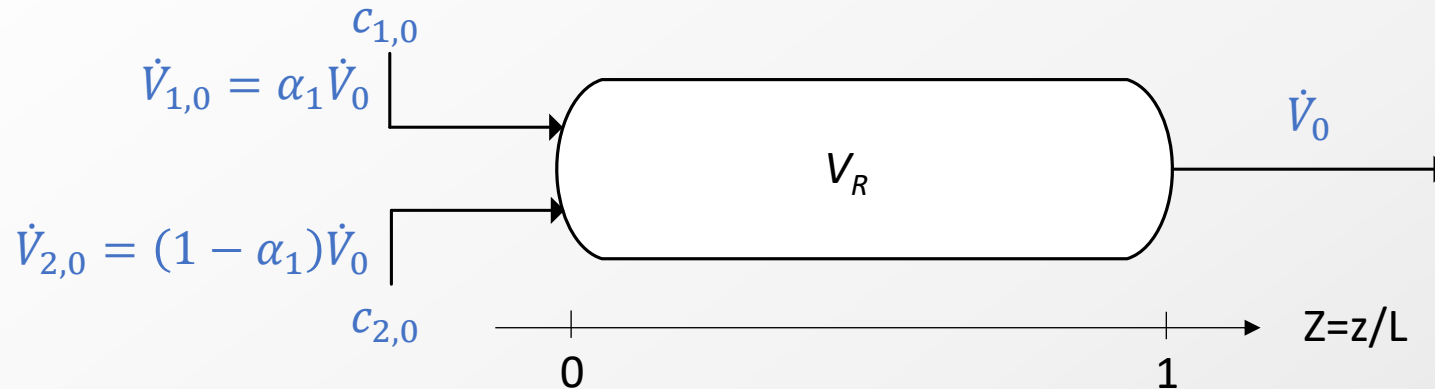
$$\bar{c}_2 = \bar{c}_{2,0} - \bar{c}_{1,0} + \bar{c}_1$$

$$f_1 = \frac{\bar{c}_1}{\bar{c}_{1,0}} = (1 - X_1)$$

$$M = \frac{\bar{c}_{2,0}}{\bar{c}_{1,0}}$$

$$DaI = k\bar{c}_{1,0}\tau_{PFR}$$

Effect of segregation intensity in PFR



$$\frac{df_1}{dZ} = -DaI[f_1(f_1 + M - 1) - I_s M]$$

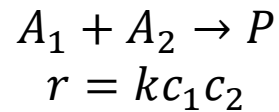
$$I_s = \exp\left(-\frac{\tau_{PFR}}{t_{mx}} Z\right) = \exp\left(-\frac{DaI}{DaI_{mx}} Z\right)$$

$$DaI_{mx} = \frac{t_{mx}}{t_r} = k\bar{c}_{1,0}t_{mx}$$

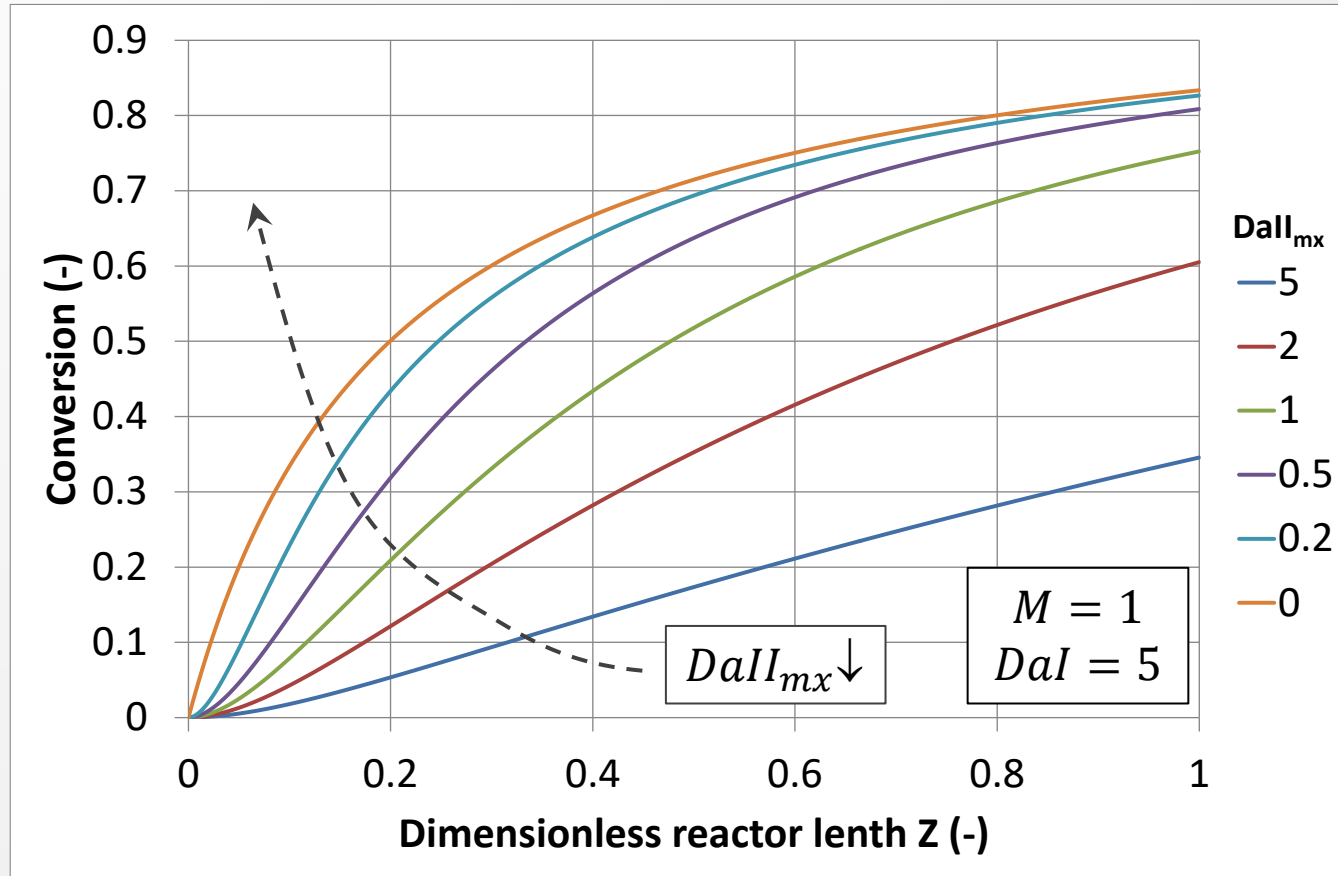
$$\frac{df_1}{dZ} = -DaI \left[f_1(f_1 + M - 1) - \exp\left(-\frac{DaI}{DaI_{mx}} Z\right) M \right]$$

→ Numerical integration

Effect of segregation intensity in PFR

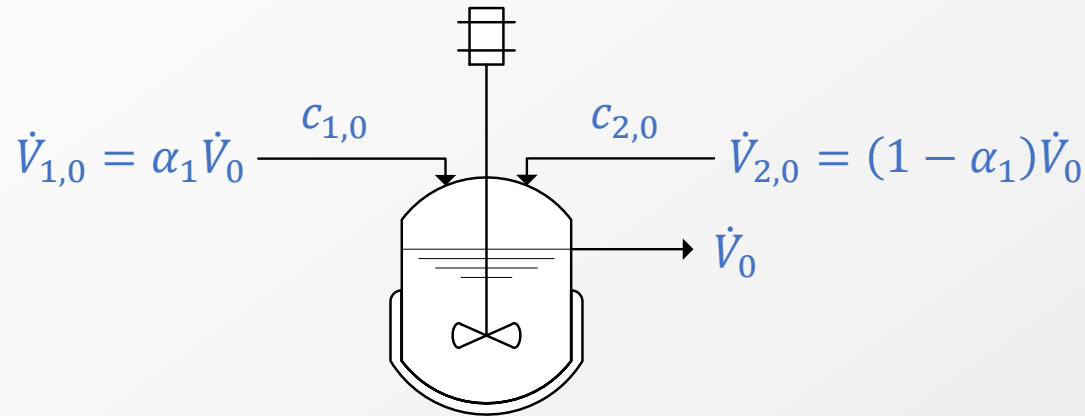


$$DaII_{mx} = \frac{t_{mx}}{t_r} = k\bar{c}_{1,0}t_{mx}$$



→ Higher conversion when mixing time ($DaII_{mx}$) decreases

Effect of segregation intensity in ideal CSTR



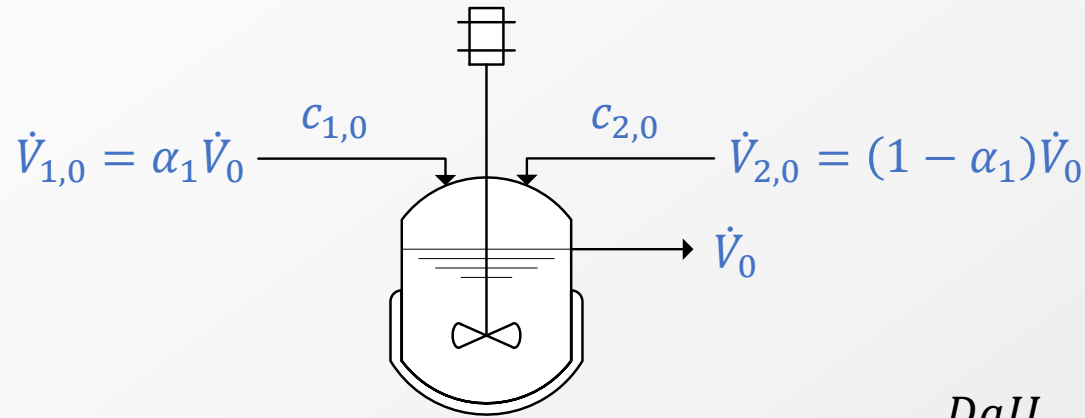
CSTR: mass balance for component A_1 :

$$\bar{c}_1 = \bar{c}_{1,0} + \tau_{CSTR} \bar{R}_1 = \bar{c}_{1,0} - k \tau_{CSTR} (\bar{c}_1 \bar{c}_2 - I_s \bar{c}_{1,0} \bar{c}_{2,0})$$

$$X_1 = \frac{\left(1 + M + \frac{1}{DaI}\right) - \sqrt{\left(1 + M + \frac{1}{DaI}\right)^2 - 4M(1 - I_s)}}{2}$$

$$\left\{ \begin{array}{l} \bar{c}_2 = \bar{c}_{2,0} - \bar{c}_{1,0} + \bar{c}_1 \\ M = \frac{\bar{c}_{2,0}}{\bar{c}_{1,0}} \\ DaI = k \bar{c}_{1,0} \tau_{CSTR} \\ X_1 = 1 - \frac{\bar{c}_1}{\bar{c}_{1,0}} \end{array} \right.$$

Effect of segregation intensity in ideal CSTR



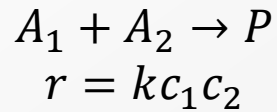
$$DaI_{mx} = \frac{t_{mx}}{t_r} = k\bar{c}_{1,0}t_{mx}$$

$$X_1 = \frac{\left(1 + M + \frac{1}{DaI}\right) - \sqrt{\left(1 + M + \frac{1}{DaI}\right)^2 - 4M(1 - I_s)}}{2}$$

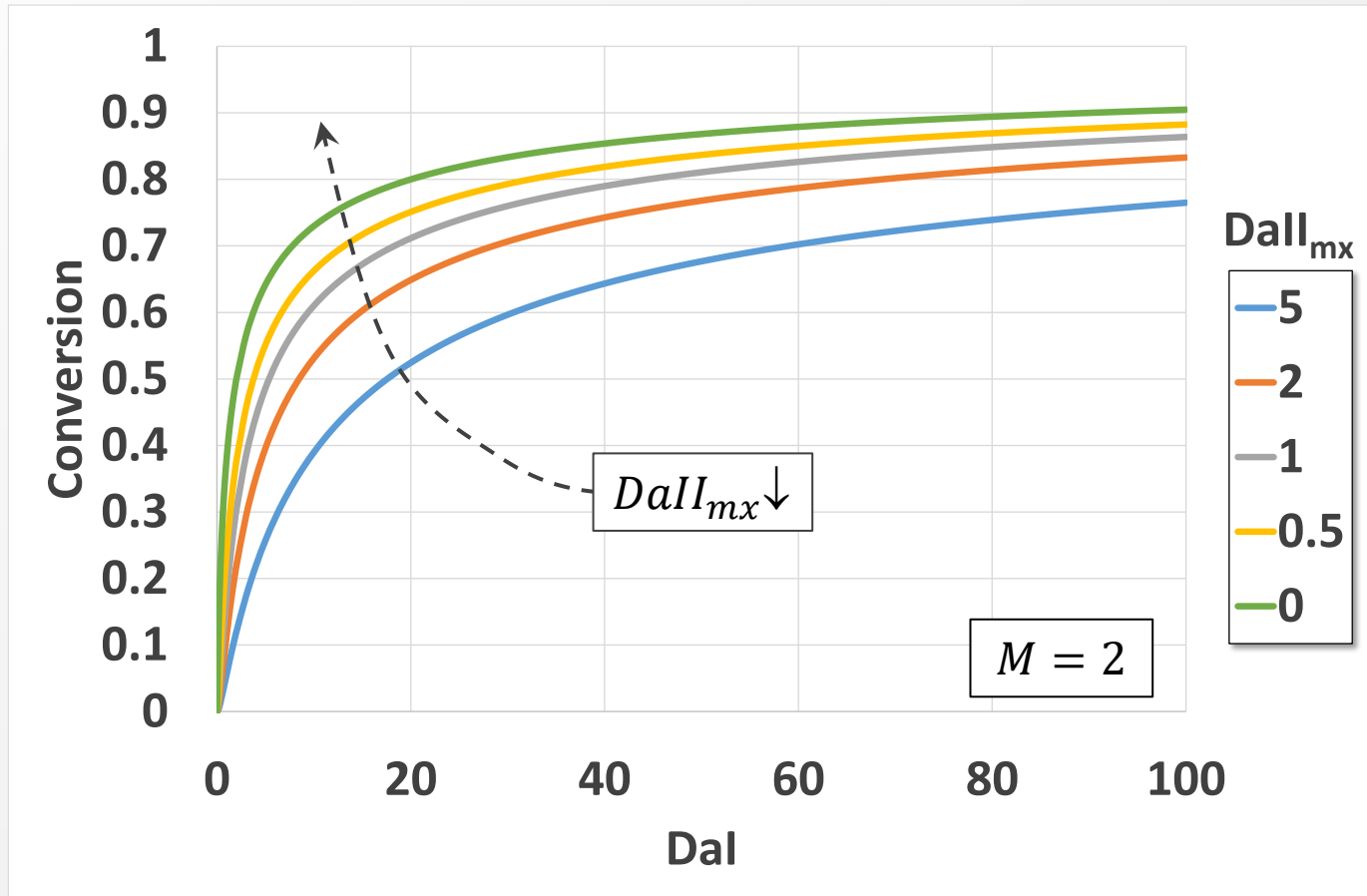
$$I_s = \frac{1}{1 + \frac{\tau_{CSTR}}{t_{mx}}} = \frac{1}{1 + \frac{DaI}{DaI_{mx}}}$$

$$X_1 = \frac{\left(1 + M + \frac{1}{DaI}\right) - \sqrt{\left(1 + M + \frac{1}{DaI}\right)^2 - 4M\left(\frac{\frac{DaI}{DaI_{mx}}}{1 + \frac{DaI}{DaI_{mx}}}\right)}}{2}$$

Effect of segregation intensity in ideal CSTR

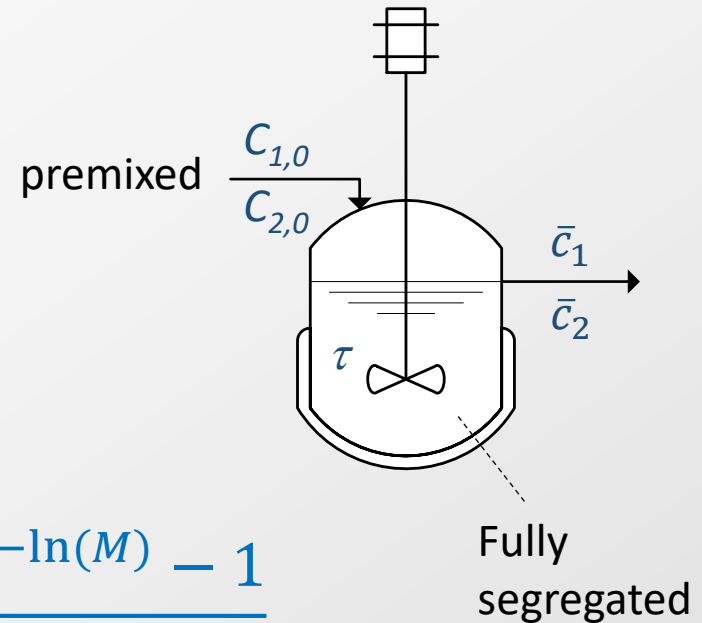
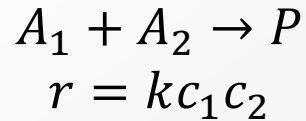


$$DaII_{mx} = \frac{t_{mx}}{t_r} = k\bar{c}_{1,0}t_{mx}$$



→ Higher conversion when mixing time ($DaII_{mx}$) decreases

Conversion in fully segregated CSTR with premixed feed



Mean conversion: $\bar{X}_1 = \int_0^\infty X_1(t) E(t) dt$

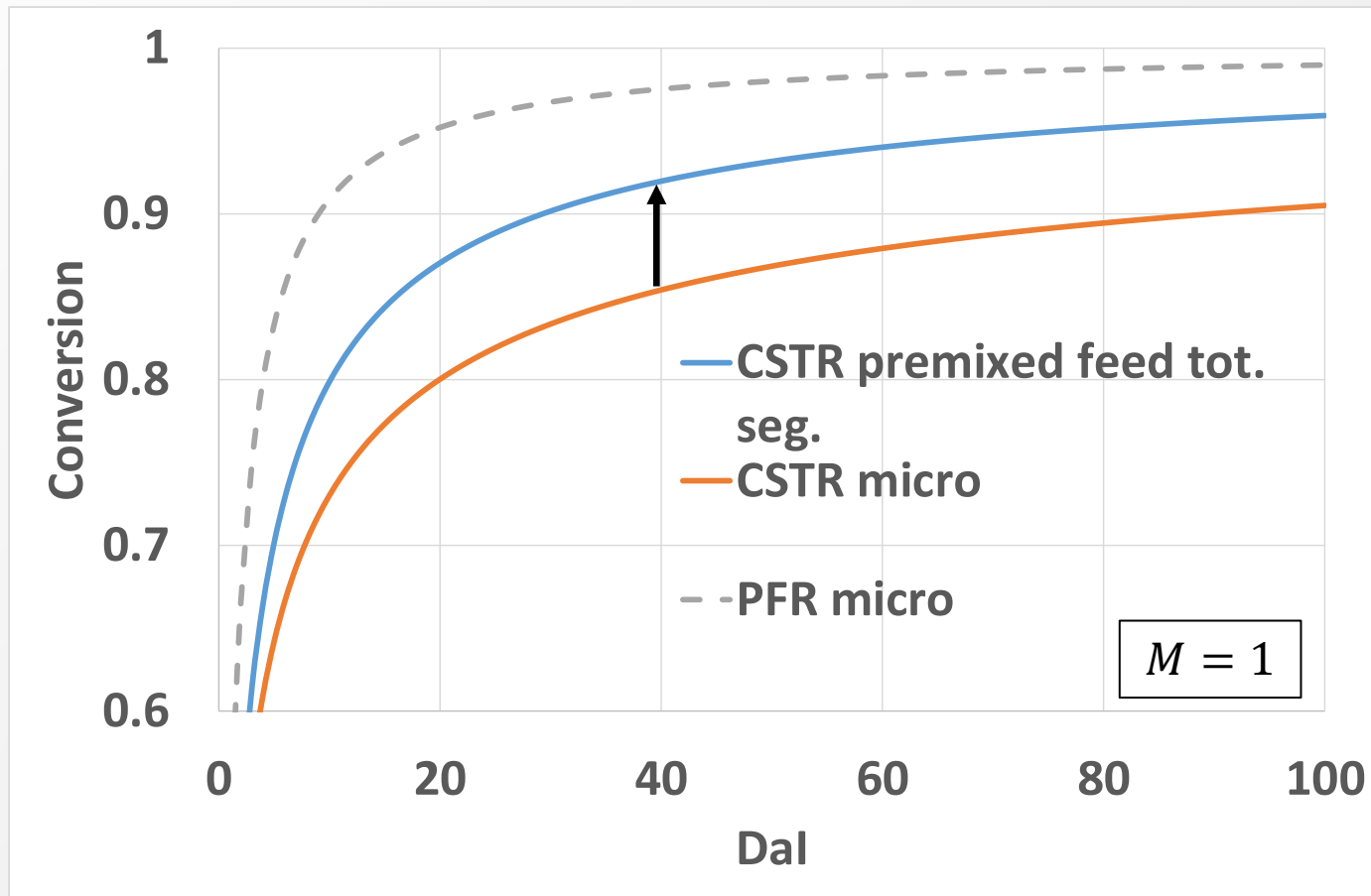
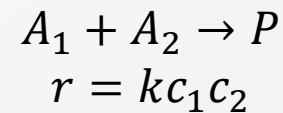
Kinetics:
$$X_1(t) = \frac{M e^{kc_{1,0}t(1-M) - \ln(M)} - 1}{e^{kc_{1,0}t(1-M) - \ln(M)} - 1}$$

RTD:
$$E(t) = \frac{1}{\tau} e^{-t/\tau}$$

Mean conversion:
$$\bar{X}_1(t) = \frac{1}{\tau} \int_0^\infty \frac{M e^{kc_{1,0}t(1-M) - \ln(M)} - 1}{e^{kc_{1,0}t(1-M) - \ln(M)} - 1} e^{-t/\tau} dt$$

→ Numerical integration

Fully segregated CSTR with premixed feed vs micromixed PFR and CSTR

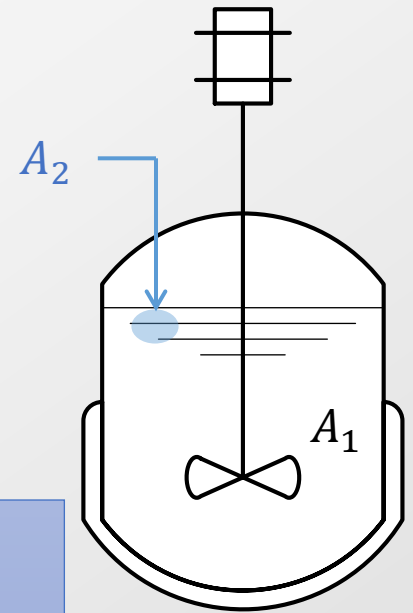
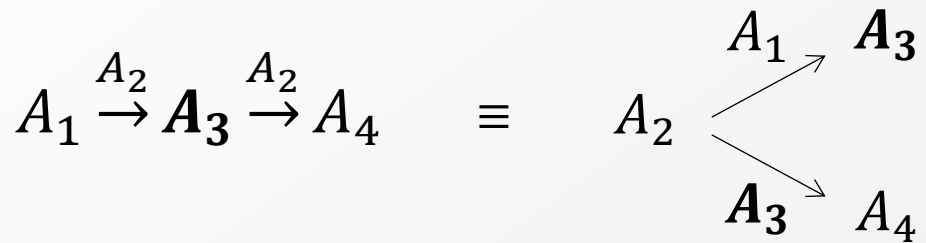


→ Higher conversion in fully segregated CSTR with pre-mixed feed vs micromixed CSTR

Effect of segregation intensity on performance and selectivity of consecutive-competitive reactions



Effect of segregation on product distribution (qualitative)



Slow mixing of A_1 and A_2 :

- Formation of zones with excess A_2
- A_1 in these A_2 -rich zones is quickly converted to A_3
- A_2 is still present in the zones and, for lack of A_1 (which cannot be transported to the zone fast enough because of the slow mixing process), reacts with A_3 to form A_4)

Effect of segregation intensity

Modified rate equations $A_1 \xrightarrow{A_2} A_3 \xrightarrow{A_2} A_4$

$$A_1 + A_2 \xrightarrow{k_1} A_3 \quad r_1 = k_1 \overline{c_1 c_2} = k_1 [\bar{c}_1 \bar{c}_2 + cov(c_1 c_2)]$$

$$A_3 + A_2 \xrightarrow{k_2} A_4 \quad r_2 = k_2 \overline{c_3 c_2} \cong k_2 \bar{c}_3 \bar{c}_2 \quad (cov(c_3 c_2) \cong 0)^*$$

$$\left\{ \begin{array}{l} \bar{R}_1 = -r_1 = -k_1 [\bar{c}_1 \bar{c}_2 + cov(c_1 c_2)] \\ \bar{R}_2 = -r_1 - r_2 = -k_1 [\bar{c}_1 \bar{c}_2 + cov(c_1 c_2)] - k_2 \bar{c}_3 \bar{c}_2 \\ \bar{R}_3 = r_1 - r_2 = k_1 [\bar{c}_1 \bar{c}_2 + cov(c_1 c_2)] - k_2 \bar{c}_3 \bar{c}_2 \end{array} \right.$$

where: $cov(c_1 c_2) = I_s \cdot cov(c_1 c_2)_0 = -I_s \cdot \bar{c}_{1,0} \bar{c}_{2,0}$

Conc. of A_4 obtained from the mass balance: $\bar{c}_4 = \bar{c}_{2,0} - \bar{c}_2 - \bar{c}_3$

*Bourne, J.R. and Toor, H.L. (1977), *AIChE J.*, 23 (4), 602–604

Effect of segregation intensity

Application to PFR $A_1 \xrightarrow{A_2} A_3 \xrightarrow{A_2} A_4$

PFR: infinitesimal mass balance for component i \Rightarrow
$$-\frac{d\bar{c}_i}{dZ} = \tau_{PFR} \bar{R}_i$$

Numerical integration

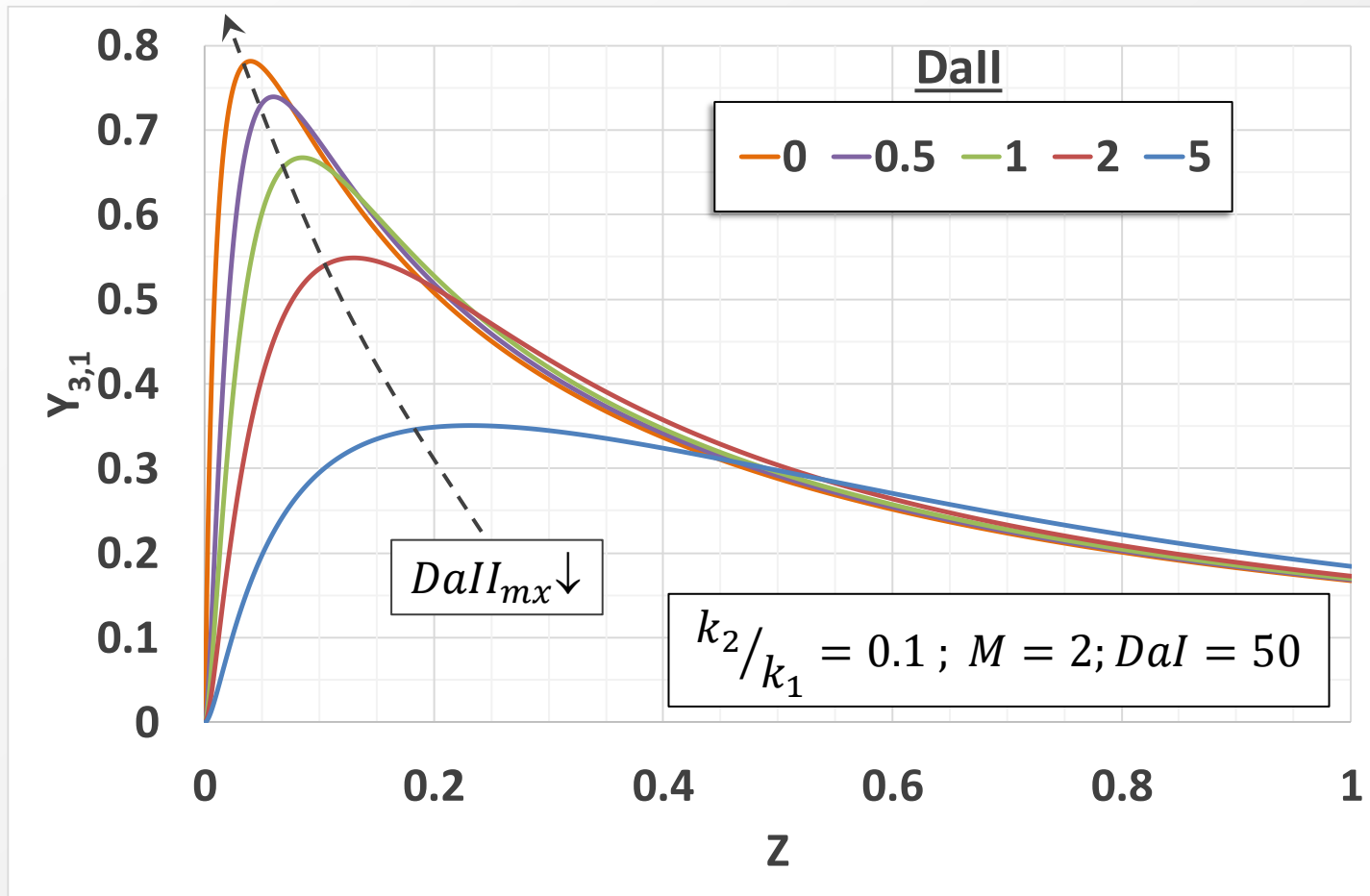
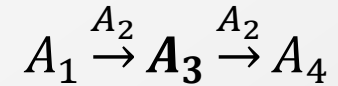
$$\left\{ \begin{aligned} -\frac{d\bar{c}_1}{\tau_{PFR} dZ} &= -k_1 [\bar{c}_1 \bar{c}_2 - I_s \cdot \bar{c}_{1,0} \bar{c}_{2,0}] = -k_1 \left[\bar{c}_1 \bar{c}_2 - \exp\left(-\frac{DaI}{DaII_{mx}} Z\right) \bar{c}_{1,0} \bar{c}_{2,0} \right] \\ -\frac{d\bar{c}_2}{\tau_{PFR} dZ} &= -k_1 \left[\bar{c}_1 \bar{c}_2 - \exp\left(-\frac{DaI}{DaII_{mx}} Z\right) \bar{c}_{1,0} \bar{c}_{2,0} \right] - k_2 \bar{c}_3 \bar{c}_2 \\ -\frac{d\bar{c}_3}{\tau_{PFR} dZ} &= k_1 \left[\bar{c}_1 \bar{c}_2 - \exp\left(-\frac{DaI}{DaII_{mx}} Z\right) \bar{c}_{1,0} \bar{c}_{2,0} \right] - k_2 \bar{c}_3 \bar{c}_2 \end{aligned} \right.$$

$$\bar{c}_4 = \bar{c}_{2,0} - \bar{c}_2 - \bar{c}_3$$

$$DaI = k_1 \bar{c}_{1,0} \tau_{PFR}$$

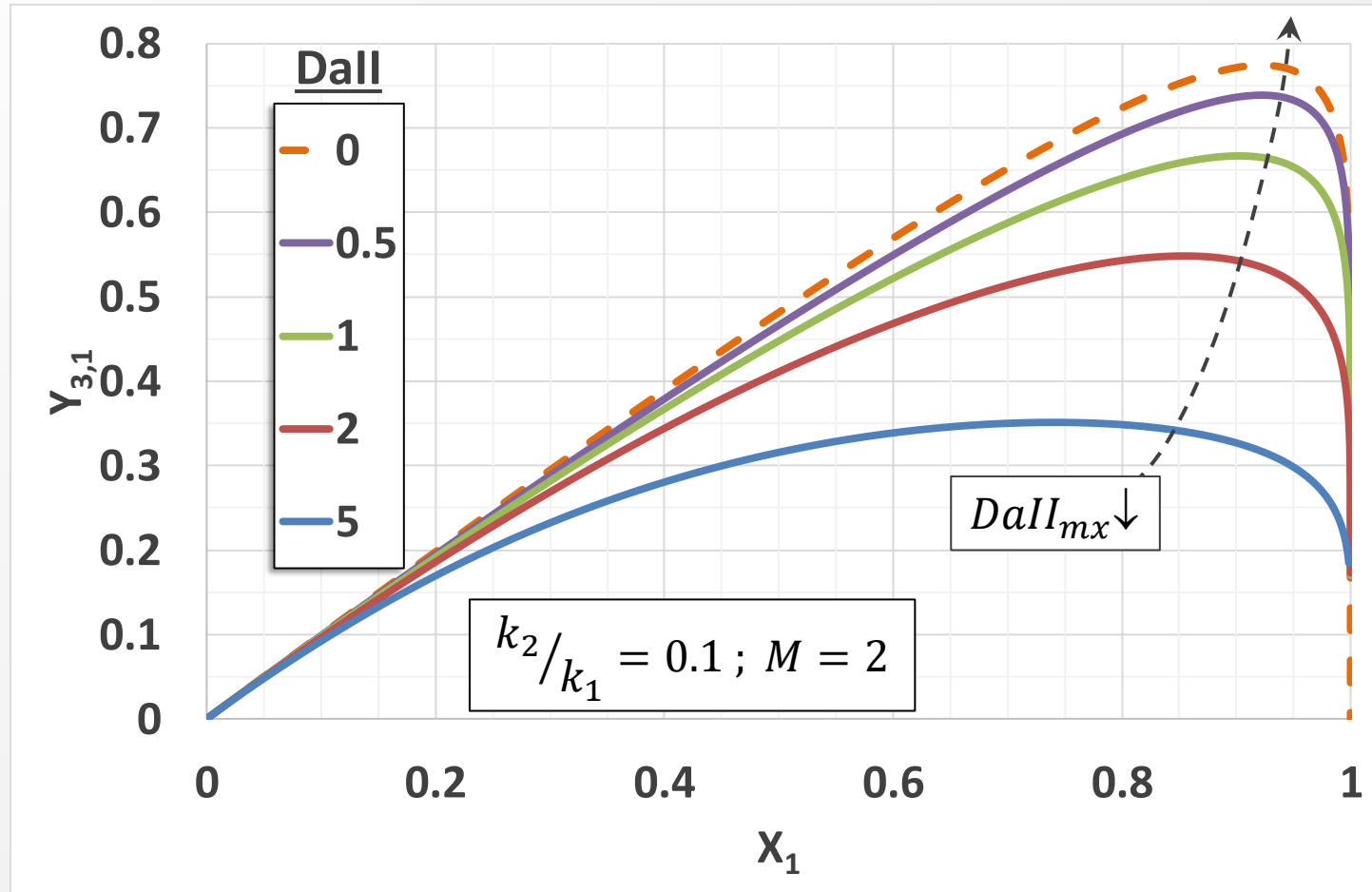
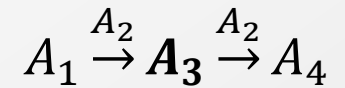
$$DaII_{mx} = k_1 \bar{c}_{1,0} t_{mx}$$

Effect of segregation intensity on intermediate product yield (PFR)



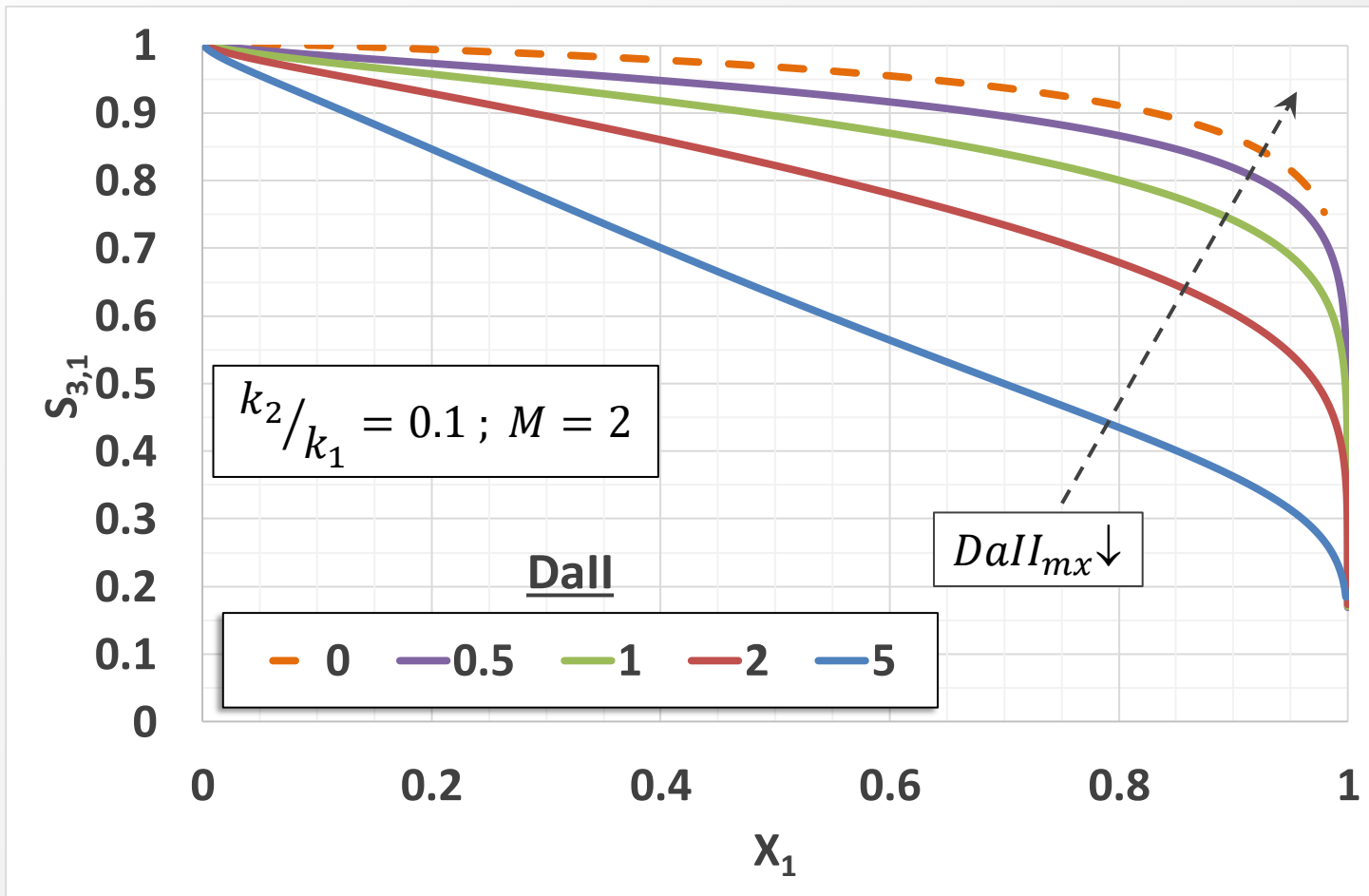
→ Higher yield when mixing time ($DaII_{mx}$) decreases

Effect of segregation intensity on intermediate product yield (PFR)



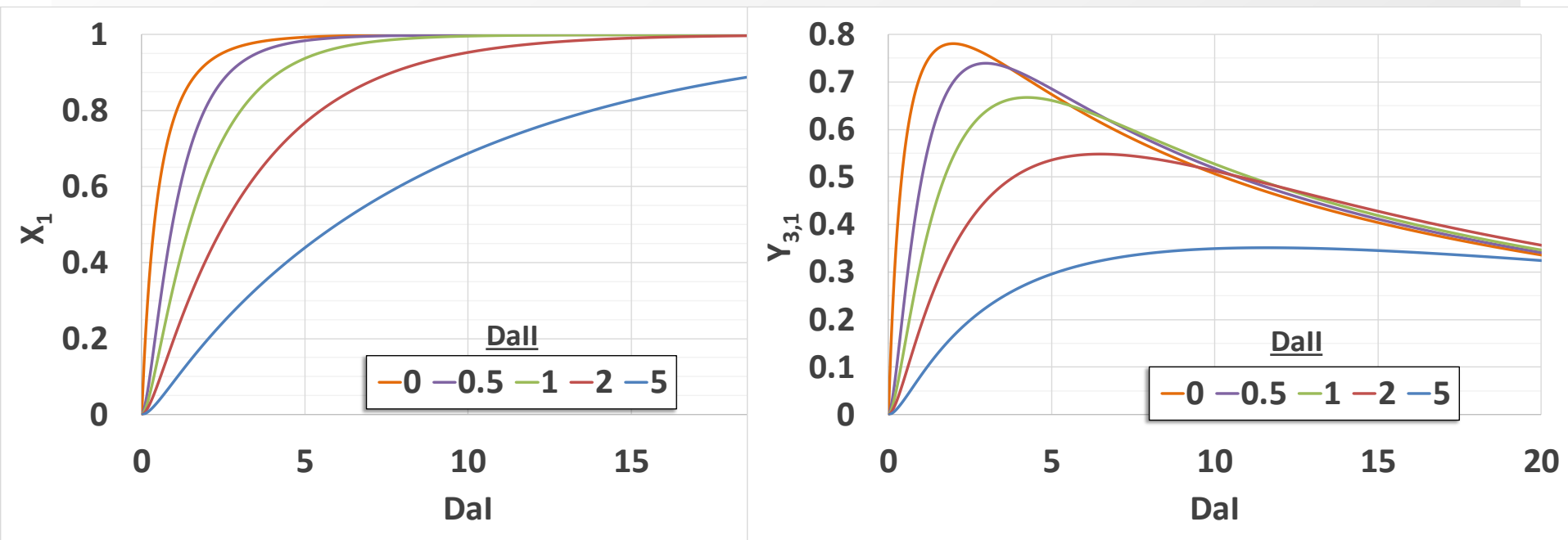
→ Higher yield when mixing time ($DaII_{mx}$) decreases

Effect of segregation intensity on intermediate product selectivity (PFR) $A_1 \xrightarrow{A_2} A_3 \xrightarrow{A_2} A_4$



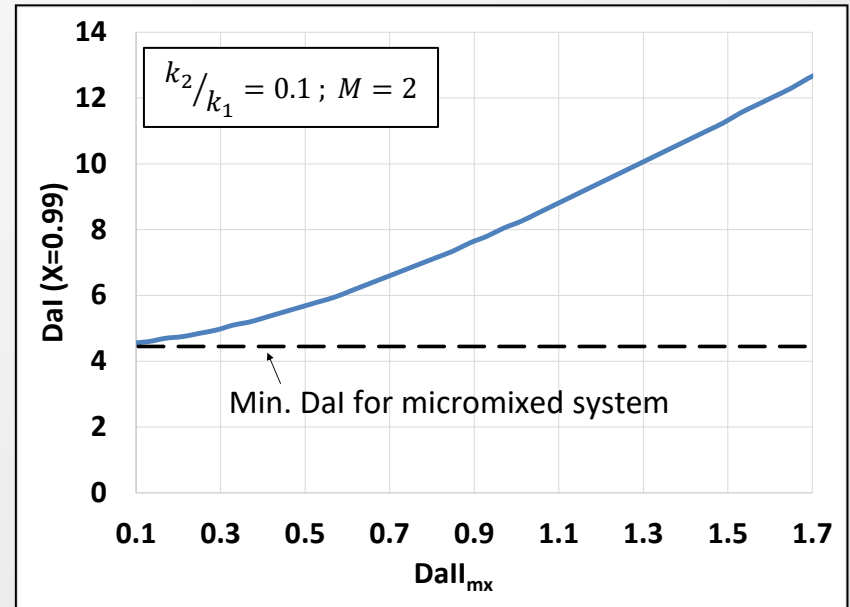
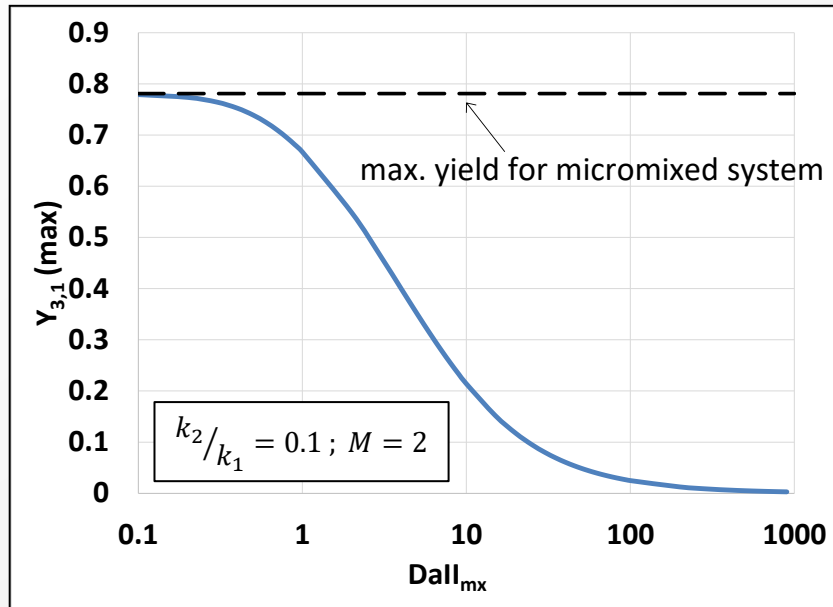
→ Higher selectivity when mixing time ($DaII_{mx}$) decreases

Effect of segregation intensity on conversion and intermediate product yield (PFR) $A_1 \xrightarrow{A_2} A_3 \xrightarrow{A_2} A_4$



→ Higher conversion and higher yield when mixing time ($Da_{II_{mx}}$) decreases

Effect of segregation intensity on performance and selectivity (PFR) $A_1 \xrightarrow{A_2} A_3 \xrightarrow{A_2} A_4$

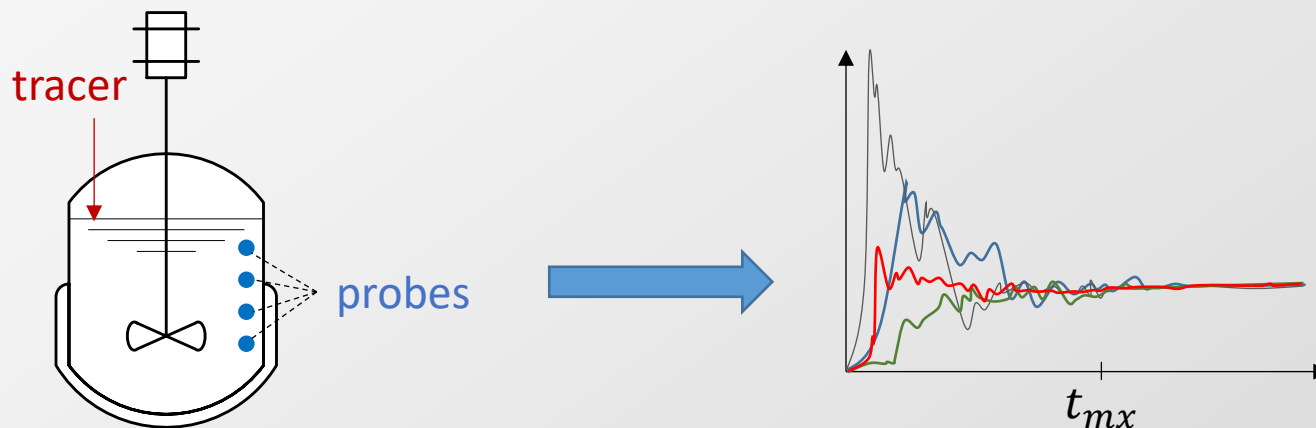


- Higher maximum yield when mixing time ($DaII_{mx}$) decreases
- Lower DaI required to reach 99% conversion when mixing time ($DaII_{mx}$) decreases
- Negligible effect of segregation when $t_{mx} \leq 0.1 t_r$ (i.e., $DaII_{mx} \leq 0.1$)

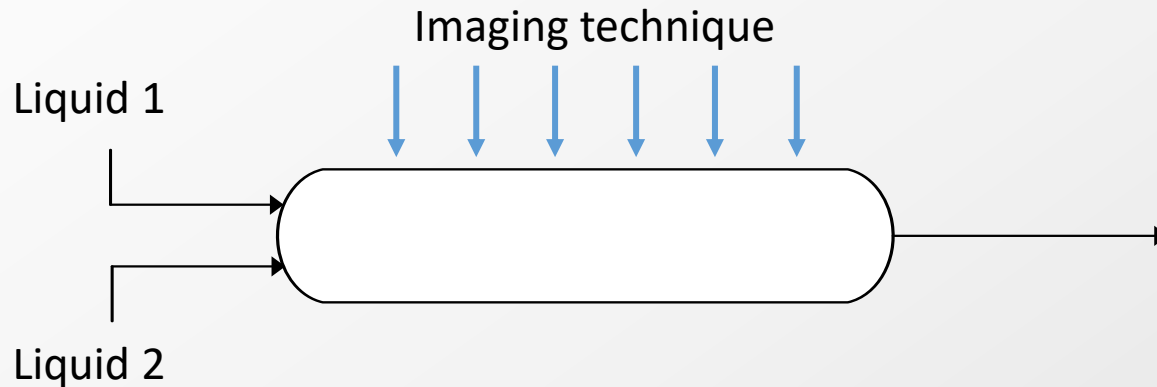
Experimental mixing characterization

Physical methods – batch reactor

- Inject inert tracer into reactor
- Monitor tracer concentration or physical property (electrical conductivity, pH, color, temperature, etc.) vs time in key reactor locations
- Mixing time = time to reach 'uniformity' in the reactor
 - many definitions of degree of uniformity exist (t_{m95} , t_{m99} , etc.)
 - t_{mx} is a function of the definition



Physical methods – continuous system : dilution method



- Dilution method: liquid 1 is transparent, liquid 2 contains a dye
- Requires transparent device (or viewing window) and imaging technique (video camera or high-speed camera) to visualize spreading of dye along flow axis
- Visualization of color spreading along the mixer \Rightarrow mixing is judged complete when the concentration of the dye is uniform across the channel cross section
- More precise method: measurement of conc. variance decay (segregation) along reactor to estimate mixing time. Requires measurement of the local concentration in a controlled volume and express its fluctuation with respect to the mean.

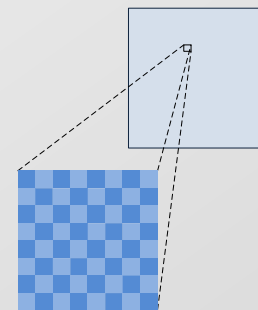
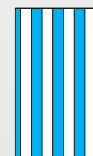
Physical methods – alternative methods



- Induced color change (pH change for example)
 - Allows to visualize interfaces (reactions occurring at the interface)
- Change in electrical conductivity

Physical methods – drawbacks

- Need for transparent mockup (glass or polymeric)
- Visualization of the mixing process is perpendicular to the flow
 - Images provide information averaged over the depth of the microchannel
 - Difference between a perfectly mixed system and a multi-layered system cannot be distinguished (optical artifact)
- Degree of mixing depends on spatial resolution of the probe used to measure local concentration
 - Use of fluorescent dye (fluorescein or rhodamin) with confocal scanning microscopy allows distinguishing between perfect mixing and multi-lamellae by providing 3D imaging of the flow
- Physical methods inadequate if sampling volume is larger than smallest segregation scales



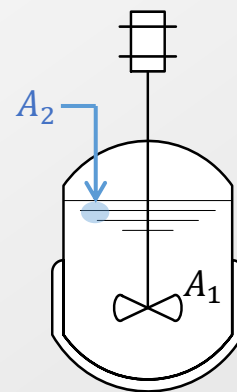
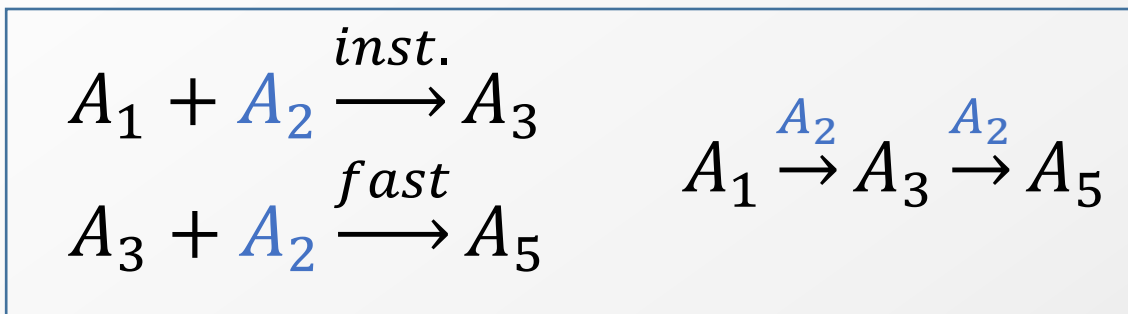
⇒ Chemical methods offer a solution to this major drawback

Chemical methods

- Use of fast, competitive chemical reactions that are strongly influenced by mixing
- **Offline** measurement of the product distribution to characterize the mixing efficiency
- When t_r and t_{mx} are of the same order of magnitude, consumption rate is lower than intrinsic reaction rate
- Reagent concentrations need to be adjusted to tune reaction rates
- Models for the coupling of mixing and reaction are used to estimate t_{mx} (e.g., Interaction by Exchange with the Mean or Engulfment models)

Chemical methods

Consecutive competitive reactions

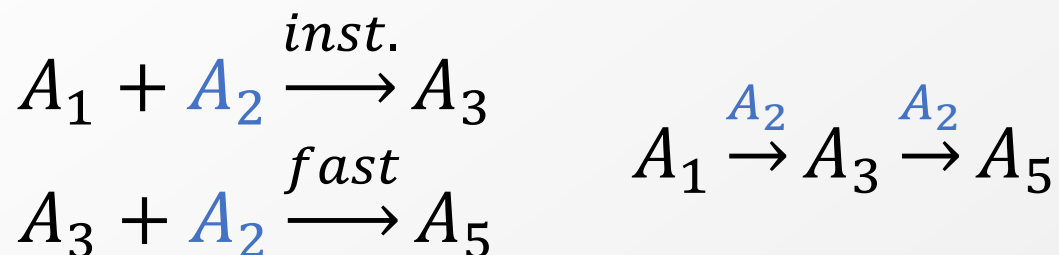


- Reaction 1 is instantaneous (e.g., acid-base) $t_{r1} \ll t_{r2}$
- Reaction 2 is 'tuned' so that $t_{r2} \cong t_{mx}$
- Stoichiometric defect of A_2 relatively to A_1
- Fast mixing ($t_{mx} \ll t_{r2}$) \Rightarrow only A_3 formed
- Slow mixing (t_{mx} close to t_{r2}) \Rightarrow A_5 is also formed
- Conc. of A_5 increases if mixing slows down

Chemical methods

Consecutive competitive reactions

Definition of segregation index (X_S)*

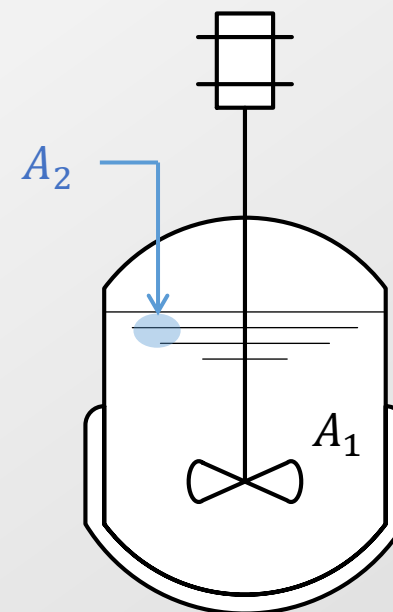


$$X_S = \frac{\text{Reagent consumption to } A_5}{\text{Total reagent consumption}} = \frac{2c_5}{2c_5 + c_3}$$

$$0 < X_S < 1$$

$X_S = 0$: micromixed system

$X_S = 1$: fully segregated system

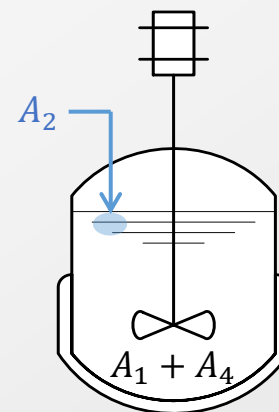
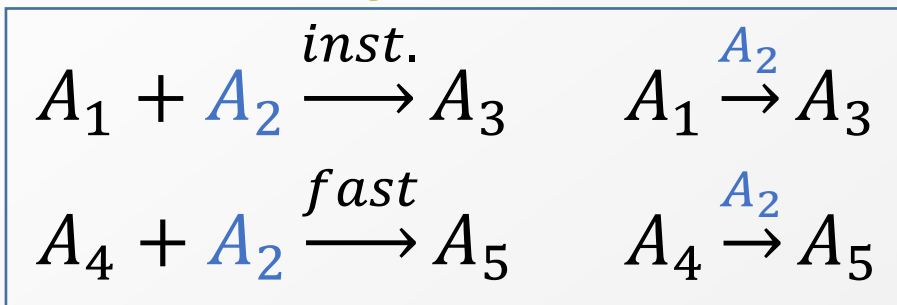


- Overall, $\bar{c}_2 < \bar{c}_1$
- Locally, if slow mixing $\Rightarrow c_2 > c_1 \Rightarrow$ formation of A_5

*Bourne, Kozicki, Rys, 1981

Chemical methods

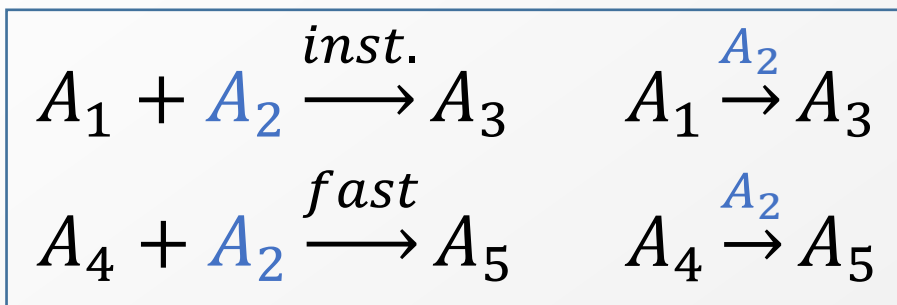
Parallel competitive reactions



- Reaction 1 is instantaneous (e.g., acid-base) $t_{r1} \ll t_{r2}$
- Reaction 2 is tuned so that $t_{r2} \cong t_{mx}$
- Stoichiometric defect of A_2 relatively to $A_1 + A_4$
- Fast mixing ($t_{mx} \ll t_{r2}$) \Rightarrow only A_3 formed
- Slow mixing (t_{mx} close to t_{r2}) \Rightarrow A_5 is also formed
- Conc. of A_5 increases if mixing slows down
- t_{r2} can be independently tuned by adjusting c_4 (an advantage over the consecutive competitive method)

Chemical methods

Parallel competitive reactions



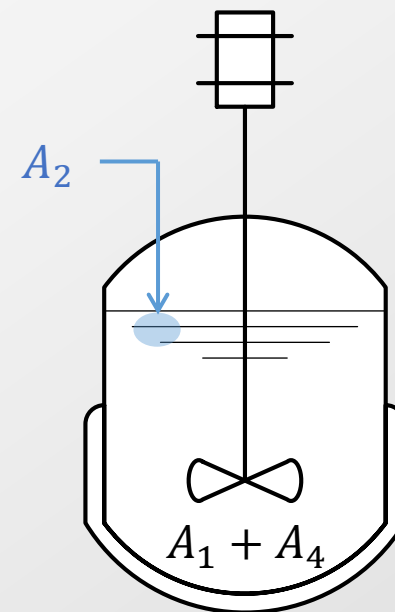
$$X_S = \frac{A_5 \text{ yield (exp.)}}{A_5 \text{ yield (complete seg.)}} = \frac{Y}{Y_{CS}} = \frac{c_5}{c_{2,0}} \frac{c_{1,0} + c_{4,0}}{c_{4,0}}$$

Micromixed conditions:

$$\frac{c_5}{c_3} = \frac{R_5}{R_3} = \frac{k_2 c_4 c_2}{k_1 c_1 c_2} = \frac{k_2 c_4}{k_1 c_1}$$

Fully segregated conditions:

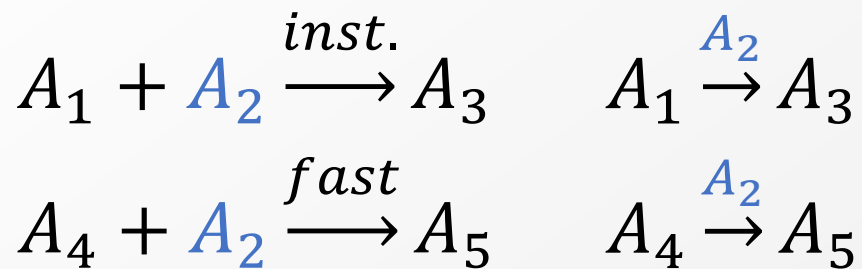
$$\frac{c_5}{c_3} = \frac{R_5}{R_3} = \frac{k_{mx} c_4}{k_{mx} c_1} = \frac{c_4}{c_1}$$



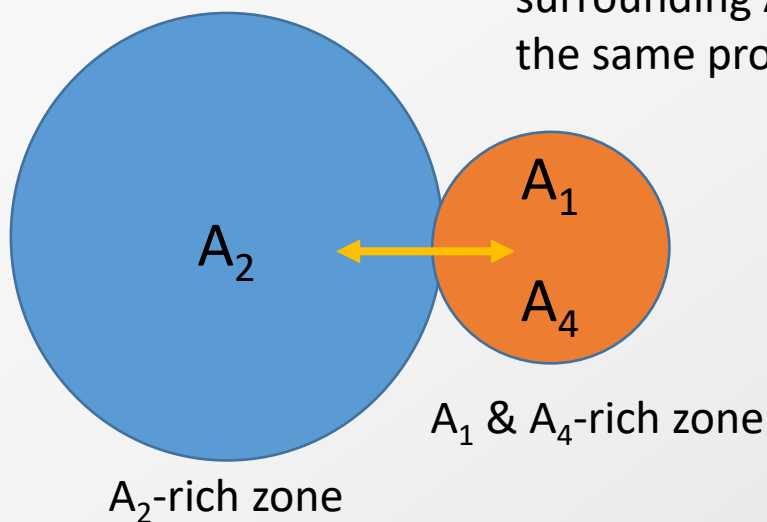
- Overall, $\bar{c}_2 < \bar{c}_1 + \bar{c}_4$
- Locally, if slow mixing $\Rightarrow c_2 > c_1 + c_4 \Rightarrow$ formation of A_5

Parallel competitive reactions

Full segregation



Full segregation \Rightarrow local excess of $A_2 \Rightarrow$ full conversion of surrounding A_1 & A_4 fluid aggregates \Rightarrow A_1 & A_4 consumed in the same proportion as in the feeds (i.e., no selectivity)

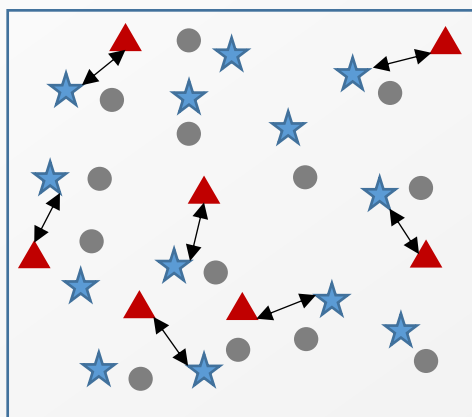
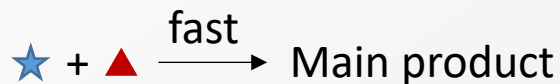


$$Y_{CS} = \frac{c_{4,0}}{c_{1,0} + c_{4,0}}$$

← A_2 converted to A_5
← Total A_2 consumed

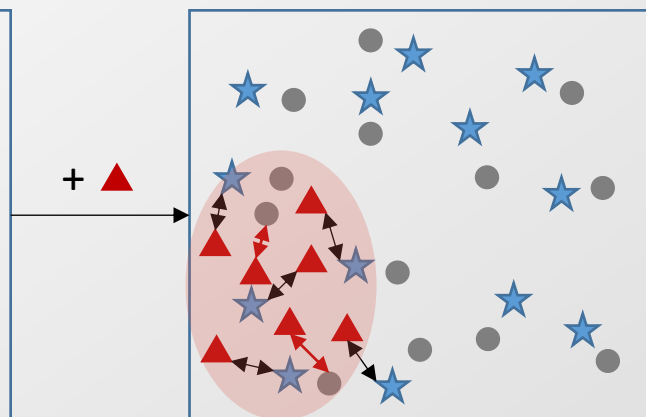
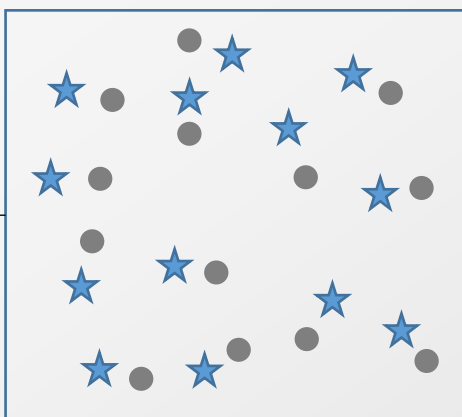
Parallel competitive reactions

Effect of mixing on by-product formation



Fast mixing

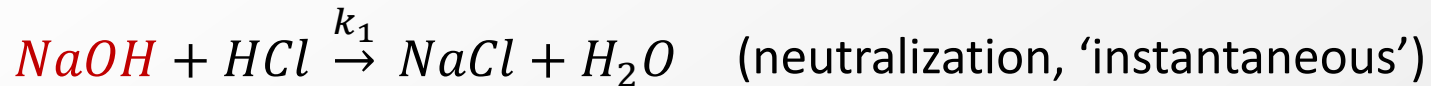
Reagent \blacktriangle well distributed into reaction mixture before reaction takes place \Rightarrow only main product formed



Slow mixing

Reaction takes place locally, before reagent \blacktriangle can be distributed into reaction mixture \Rightarrow some by-product formed

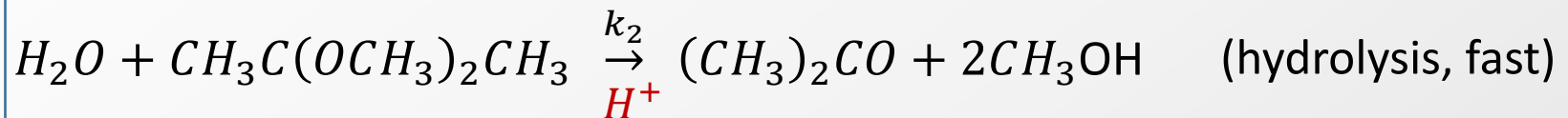
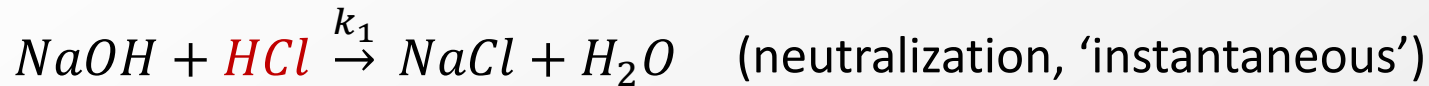
Neutralization and ester hydrolysis



- Parallel competitive reaction network
- Add NaOH solution to solution of HCl and ethyl monochloroacetate
- Well known kinetics
- Simple analysis (GC) of residual ester or ethanol formed to characterize degree of micromixing

M. Jasińska, Chemical and Process Engineering 2015, 36 (2), 171-208

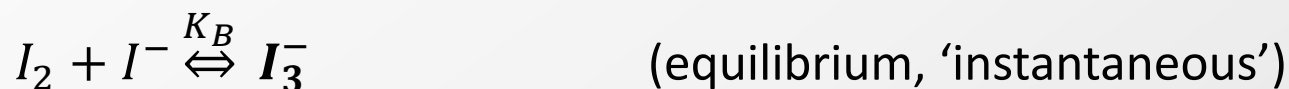
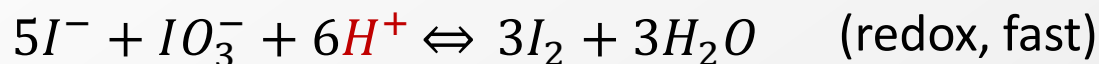
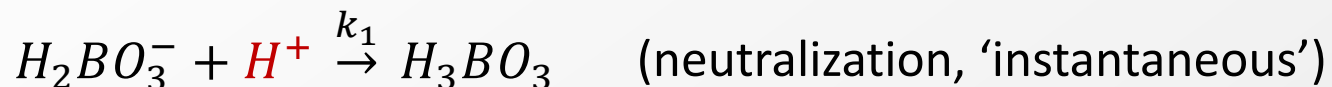
Neutralization and acetal hydrolysis



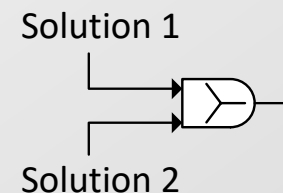
- Parallel competitive reaction network
- Add HCl solution to solution of NaOH and dimethoxypropane
- Reaction 1 irreversible, very fast with respect to mixing, moderately exothermic (-55.8 kJ/mol)
- Reaction 2 irreversible in diluted aq. solutions and acid-catalyzed.
- Ideally (micromixed system), no acetal hydrolysis \Rightarrow no methanol formed
- Analysis of methanol formed to characterize degree of micromixing (or segregation)

M. Jasińska, Chemical and Process Engineering 2015, 36 (2), 171-208

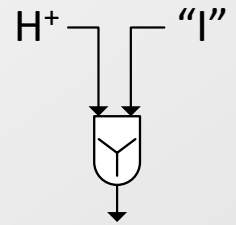
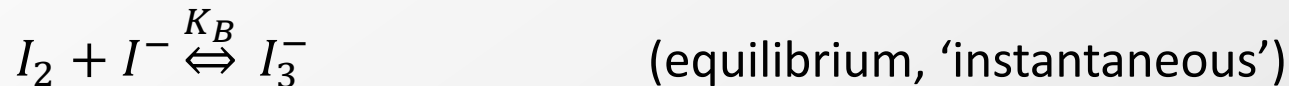
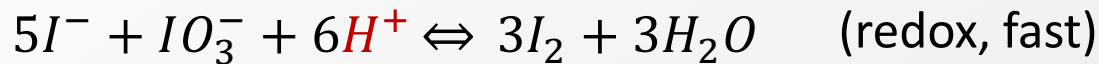
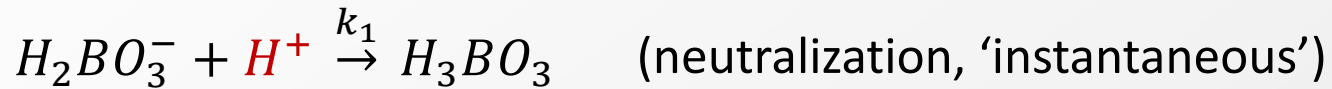
Villermaux-Dushman reaction (iodide-iodate reaction)



- Parallel competitive reaction network
- Solution 1: Sulfuric acid solution (stoichiometric defect of H^+)
- Solution 2: Iodide I^- and iodate IO_3^- in $H_2BO_3^-/H_3BO_3$ buffer
- Ideally, H^+ reacts only with $H_2BO_3^- \Rightarrow$ no I_2 formed if micromixing
- Concentration of iodine I_2 is a measure of degree of segregation
- I_2 reacts rapidly with I^- (eq. reaction, K_B well known) to yield triiodide anions I_3^-
- I_3^- easily analyzed by UV/vis spectrophotometry (353 nm)



Villermaux-Dushman reaction (iodide-iodate reaction)



- Mixing quality determined by segregation index X_S

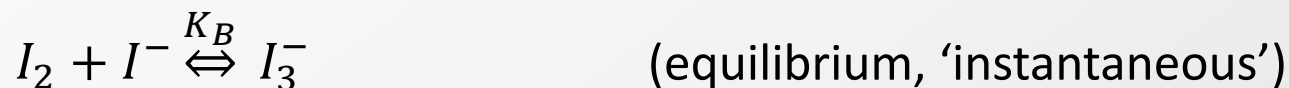
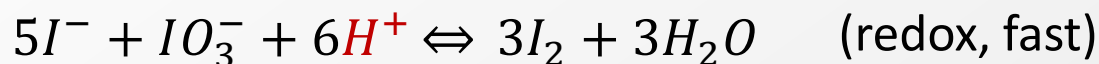
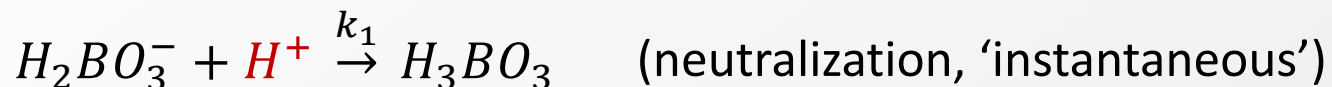
For equal volumetric flowrates
of the two solutions

$$X_S = \frac{Y}{Y_{CS}}$$

$$Y = \frac{2(\dot{n}_{I_2} + \dot{n}_{I_3^-})}{\dot{n}_{H^+,0}} = \frac{4(c_{I_2} + c_{I_3^-})}{c_{H^+,0}} \quad \left(\frac{H^+ \text{ consumed by reaction 2}}{\text{Total } H^+} \text{ experimental} \right)$$

$$Y_{CS} = \frac{6c_{IO_3^-,0}}{6c_{IO_3^-,0} + c_{H_2BO_3^-,0}} \quad \left(\frac{H^+ \text{ consumed by reaction 2}}{\text{Total } H^+} \text{ complete segregation} \right)$$

Villermaux-Dushman reaction (iodide-iodate reaction)



- Mixing quality can also be characterized by micromixedness ratio α

$$X_S = \frac{V_{CS}}{V_{CS} + V_{PM}} \rightarrow \alpha \equiv \frac{V_{PM}}{V_{CS}} = \frac{1 - X_S}{X_S}$$

V_{PM} : Perfectly mixed volume

V_{CS} : Completely segregated volume

Villermaux-Dushman reaction

Determination of c_{I_2} and $c_{I_3^-}$

$$\boxed{c_{I_3^-} = \frac{A}{\varepsilon_{353} l}} \quad \varepsilon_{353} = 2606 \text{ [} m^2 mol^{-1} \text{]} \quad (\text{Beer-Lambert})$$

$$\left. \begin{aligned} I_2 + I^- &\overset{K_B}{\rightleftharpoons} I_3^- & K_B &= \frac{c_{I_3^-}}{c_{I_2} c_{I^-}} \text{ [} l \text{ mol}^{-1} \text{]} \\ \log_{10} K_B &= \frac{555}{T} + 7.355 - 2.575 \log_{10} T \quad (T \text{ in K}) \end{aligned} \right\} \text{(Equilibrium data)}$$

$$c_{I^-} = c_{I^-,0} - \frac{5}{3} (c_{I_2} + c_{I_3^-}) - c_{I_3^-} \quad (\text{Iodine balance})$$

$$\Rightarrow -\frac{5}{3} c_{I_2}^2 + \left(c_{I^-,0} - \frac{8}{3} c_{I_3^-} \right) c_{I_2} - \frac{c_{I_3^-}}{K_B} = 0$$

Villermaux-Dushman reaction

Experimental considerations

- Adjustment of reagent concentrations to t_{mx} is necessary, as X_S is a function of t_{r1} and t_{r2} .
 - At constant concentrations (i.e., constant t_{r1} and t_{r2}), changes in X_S correctly account for variations in t_{mx}
 - X_S measured in reactors using different concentrations cannot be compared!
- Adjustment of the acid concentration to the validity range of the Beer-Lambert law for I_3^-
- The final pH should be close to 7. At lower final pH, iodine (I_2) might be formed despite perfect mixing conditions. At high pH, iodine is unstable and dissociates.

Commenges & Falk, Chem. Eng. Proc. 50
(2011) 979-990

Villermaux-Dushman reaction

Choice of concentrations

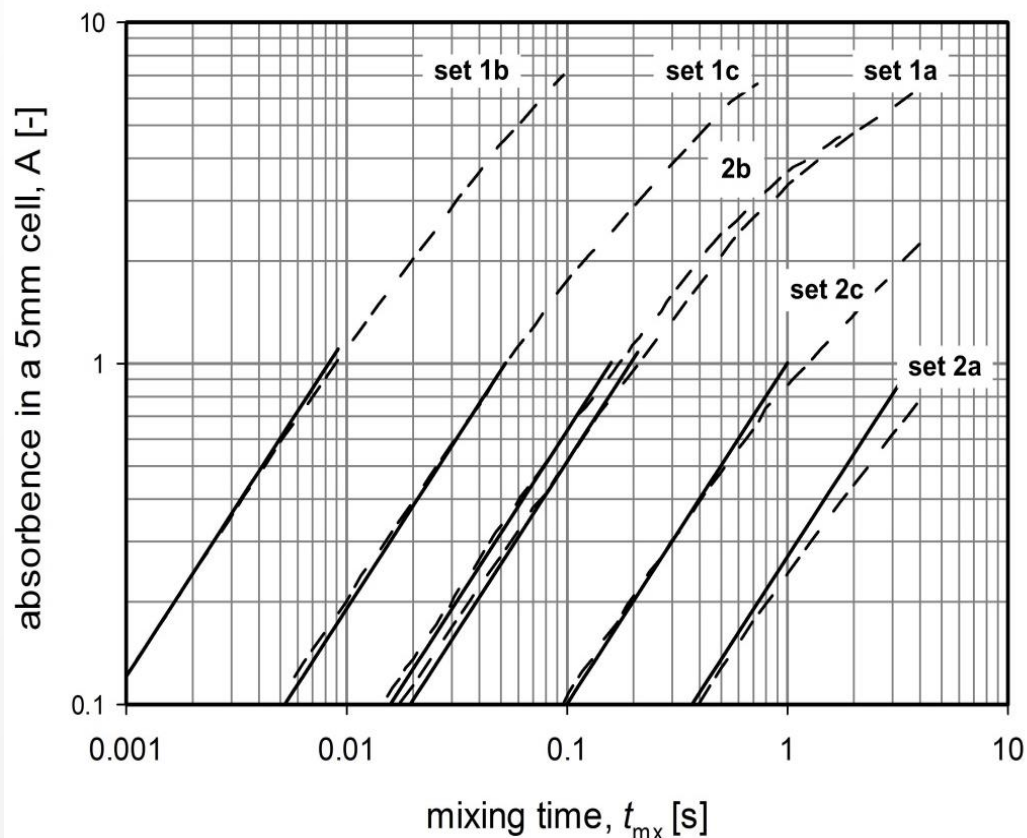
	C (mol/l)	set 1a	set 1b	set 1c	set 2a	set 2b	set 2c
Solution 1	[H ⁺]	0.03	0.06	0.04	0.015	0.03	0.02
Solution 2	[KI]	0.032	0.032	0.032	0.016	0.016	0.016
	[KIO ₃]	0.006	0.006	0.006	0.003	0.003	0.003
	[NaOH]	0.09	0.09	0.09	0.045	0.045	0.045
	[H ₃ BO ₃]	0.09	0.09	0.09	0.045	0.045	0.045

Commenges & Falk, Chem. Eng. Proc. 50
(2011) 979-990

Villermaux-Dushman reaction

Choice of concentrations

Predicted absorbance as a function of t_{mx}



Dashed lines: IEM model

Solid lines: simplified correlation (next slide)

Based on initial estimation of mixing time (see below), choose concentration set which will provide absorbance close to unity with experimental cell length (validity of Lambert-Beer law)

$$\varepsilon = \frac{Q\Delta p}{\rho V}$$

$$\varepsilon = \frac{8\nu u_m^2}{R^2} \text{ (circular geometry)}$$

$$t_{mx} \cong 0.15\varepsilon^{-0.45}$$

Commenge & Falk, Chem. Eng. Proc. 50 (2011) 979-990

Villermaux-Dushman reaction

Calculation of mixing time

Once that A has been experimentally determined, calculate t_{mx} :

$$t_{mx}[s] = 0.33 A' c_{H^+,0}^{-4.55} c_{I^-,0}^{-1.5} c_{IO_3^-,0}^{5.8} c_{NaOH,0}^{-2} c_{H_3BO_3,0}^{-2}$$

Unit of concentrations c_i : $mol \cdot l^{-1}$

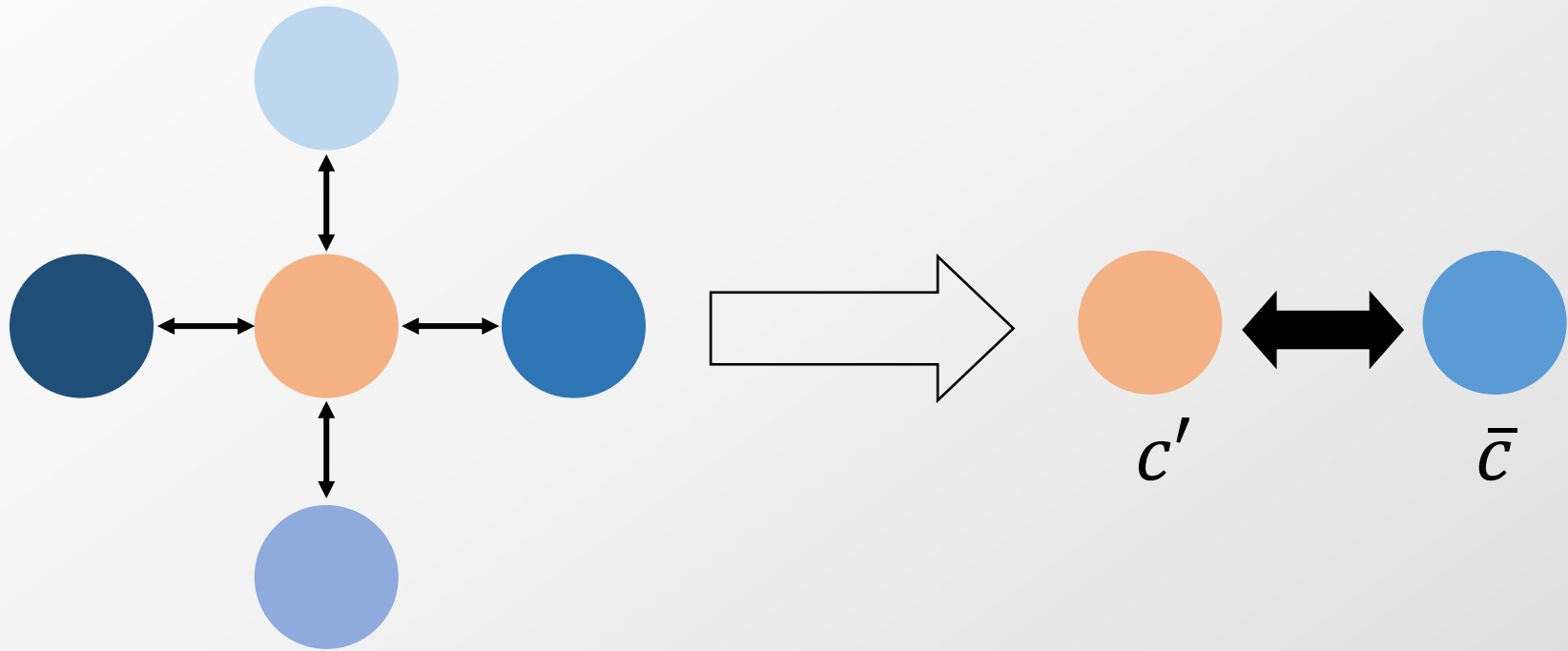
Where :

- A is the absorbance in the experimental cell of length x (mm)
- A' is the absorbance for a 1 mm optical path length
- $A' = A/x$

Commenges & Falk, Chem. Eng. Proc. 50
(2011) 979-990

IEM mixing model

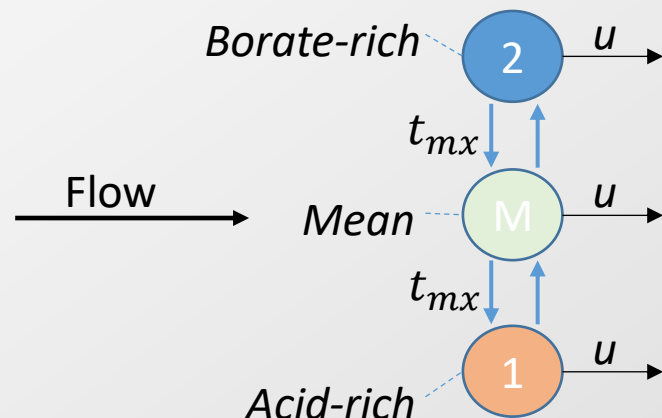
Interaction by Exchange with the Mean



Harada, 1963; Villermaux & Devillon, 1972, Costa & Trevisoi, 1972

IEM mixing model

- Interaction of mixing and reaction needs to be considered to calculate segregation index as a function of mixing time
- IEM (Interaction by Exchange with the Mean) mixing model offers a simple solution
- IEM assumptions
 - Acid-rich zone and borate-rich zones exchange mass with a characteristic time constant equivalent to the mixing time
 - Plug-flow behavior in micromixer \Rightarrow each acid-rich aggregate is in constant interaction with a borate-rich aggregate of the same age

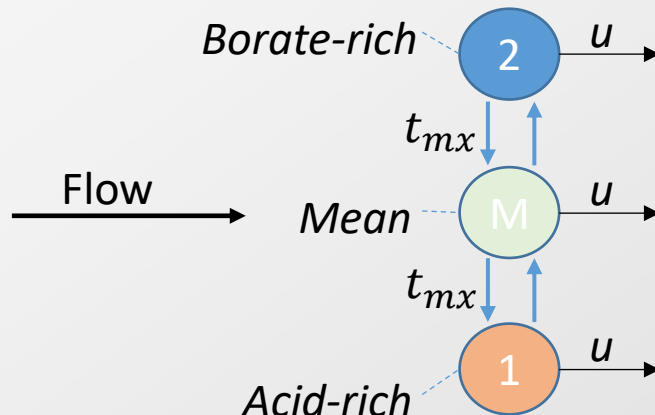


IEM mixing model

- Acid stream (subscript 1): volume fraction α
- Iodide-iodate-borate stream (subscript 2): volume fraction $(1 - \alpha)$

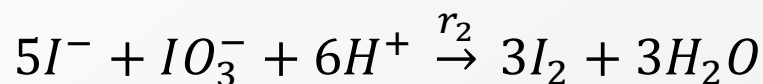
$$\frac{dc_{i,1}}{d\tau} = \frac{\bar{c}_i - c_{i,1}}{t_{mx}} + R_{i,1} \qquad \frac{dc_{i,2}}{d\tau} = \frac{\bar{c}_i - c_{i,2}}{t_{mx}} + R_{i,2}$$

$$\bar{c}_i = \alpha c_{i,1} + (1 - \alpha) c_{i,2}$$



IEM mixing model

- Kinetics of the redox reaction



$$r_2 = k_2 c_{I^-}^2 c_{IO_3^-} c_{H^+}^2 \quad k_2 \text{ is a function of the ionic strength}$$

$$I = \frac{1}{2} \sum_{i=1}^n c_i z_i^2 \quad (\text{ionic strength})$$

$$I < 0.16 \text{ mol l}^{-1} \rightarrow \log k_2 = 9.28 - 3.66\sqrt{I}$$

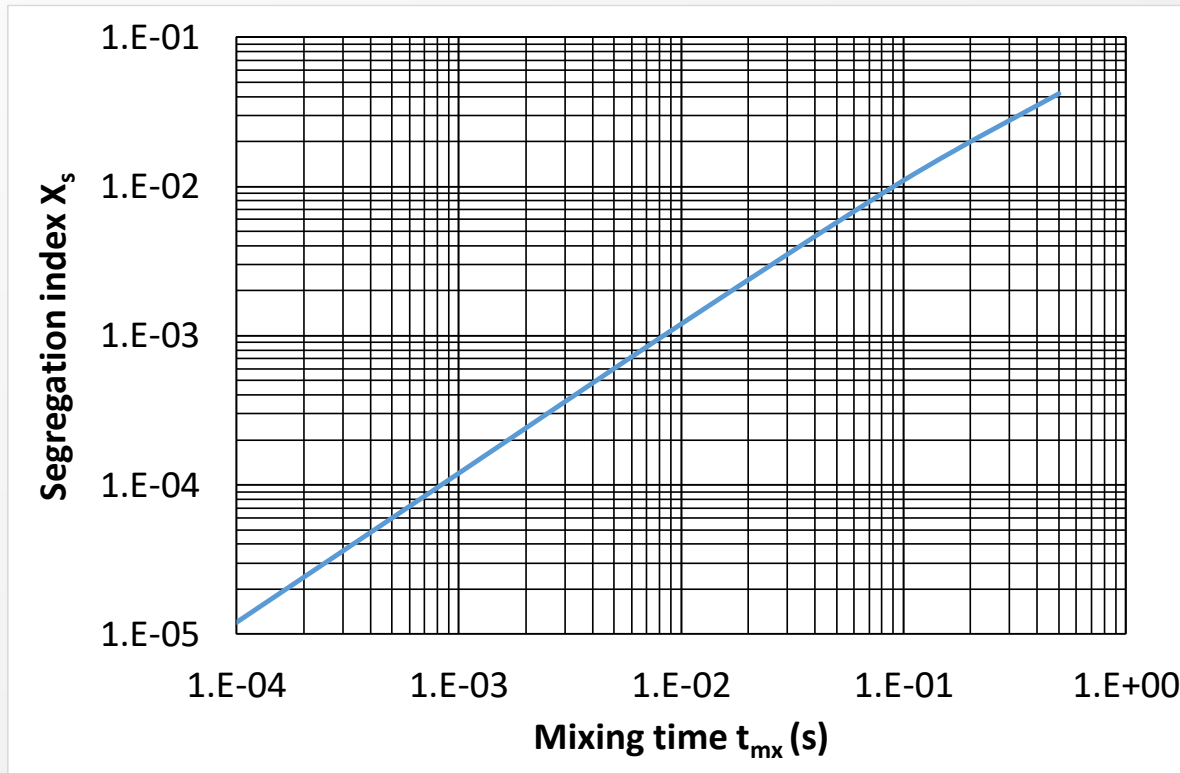
$$I > 0.16 \text{ mol l}^{-1} \rightarrow \log k_2 = 8.38 - 1.51\sqrt{I} + 0.23I$$

IEM mixing model

- Solve system of differential equations for all components i in both streams 1 and 2 using a given initial concentration set and a given value of t_{mx}
- Compute final I_2 and I_3^- concentrations at total conversion of H^+
- Calculate segregation index $X_S = f(I_2, I_3^-)$
- This provides one couple $(X_S; t_{mx})$
- Change the value of t_{mx} and repeat the procedure to obtain graph $X_S = f(t_{mx})$
- Optical density A can be calculated from I_3^- concentration (Lambert-Beer) for every t_{mx} to get graph $A = f(t_{mx})$

IEM mixing model

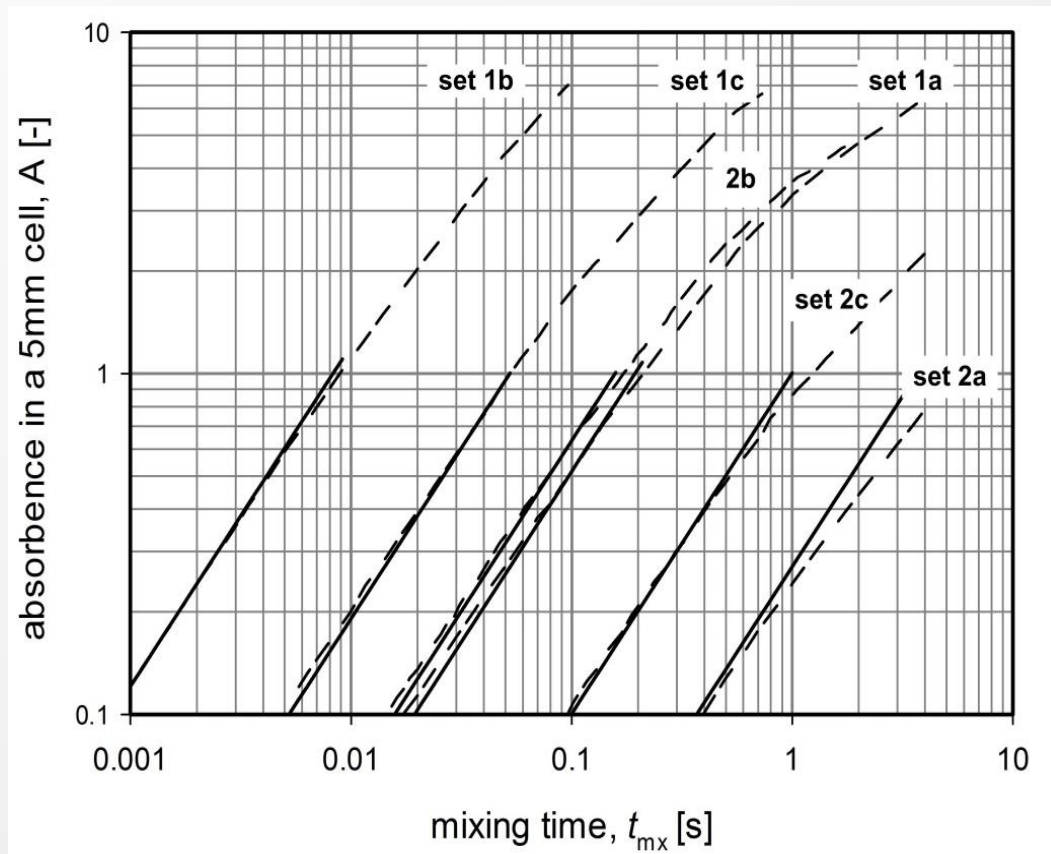
- Predicted segregation index as a function of mixing time for a given set of initial concentrations



$$c(KI) = 0.0319 \frac{\text{mol}}{\text{l}}; c(KIO_3) = 0.00635 \frac{\text{mol}}{\text{l}}; c(NaOH) = c(H_3BO_3) = 0.0898 \frac{\text{mol}}{\text{l}}; c(H_2SO_4) = 0.015 \frac{\text{mol}}{\text{l}}$$

IEM mixing model

- Predicted absorbance (5 mm optical path) as a function of mixing time for various sets of initial concentrations (dashed lines)



Solid lines: simplified correlation

Example of experimental setup for mixing characterization

